WSN Clock Synchronization by Network-coded Messages

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***Abstract*—In this paper a novel method to improve the Wireless Sensor Network synchronization process by introducing network-coded messages is proposed. The presented technique outweighs the effects of the channel's outage probability by employing the available data to refine the estimates of the pairwise clock offsets. By virtue of the proposed technique, the mean square error of the estimator is significantly reduced for a given energy budget. The claimed features are supported by extensive simulation results for sensors communicating over Rayleigh fading channels.**

***Index Terms*—clock synchronization, network coding, outage probability, wireless sensor networks.**

I. INTRODUCTION

Wireless Sensor Networks (WSN's) nodes are intrinsically designed to operate at low power and with limited processing capabilities. This allows the construction of inexpensive sensor nodes which can be deployed in large numbers. Energy management is paramount in the field of WSN. A key aspect of a WSN that enables it to achieve its endurance target is time synchronization, which allows the implementation of data fusion, transmission scheduling and energy optimization schemes through power scheduling profiles. Typically, WSN synchronization involves some form of clock offset estimation for each WSN node. Still, time synchronization leads to an inevitable energy expenditure which is exacerbated by the message loss caused by the wireless channel's outage probability [1]. It is at this stage that using network-coded (NC) timing messages to compensate for the effect of the outage probability constitutes an appealing option for enhancing the clock estimation process. A number of recent works (e.g. [2], [3]) have explored the potential of network coding theory applied to WSN in relation to energy efficiency and performance enhancement, although these have not investigated applicability of such a tool to WSN clock synchronization.

In this work we propose a novel WSN synchronization strategy that exploits network coding theory to reduce the nodes' energy expenditure, which to the best of the authors' knowledge has not been disclosed before.

II. MODEL STATEMENT

Consider a WSN consisting of N nodes denoted $u_1, u_2, \ldots, u_N$ exchanging data via wireless messages. It is customary to assume that all nodes are constructively identical, as well as the received messages are affected by equal physical-layer Additive White Gaussian Noise (AWGN) power levels, denoted as σ^2 [4, p. 37]. The wireless channel studied in this paper is considered to experience flat and fast fading, and therefore the probability of success of each message is considered statistically independent [5]. WSN nodes synchronization is attained through message exchange. Message loss occurs mainly due to the channel's outage probability, defined as the probability that the received signal power falls under a minimum acceptable threshold [1]. Let $P_{out_{ij}}$ denote the outage probability of the channel when node u_i sends message x_{ij} to node u_j. Then, node u_j will correctly receive x_{ij} with a probability equal to $1 - P_{out_{ij}}$. Traditional approaches to deal with message loss include a combination of message retransmission and an increase in the transmit power. It is well-known that energy efficiency can be attained by wisely regulating the transmit power of the sender nodes [1]. These methods, however, do not exploit the available data to devise a mechanism that outweighs the effects of the outage probability while preserving the energy and synchronization quality. The remainder of this paper focuses on providing a working solution to the aforementioned challenge.

III. CLOCK OFFSET ESTIMATION USING NETWORK CODING

The existing trade-off between number of transmitted messages and transmit power has been extensively studied in the literature [1]. Increasing the number of transmissions while reducing the transmit power is an appealing strategy since the synchronization quality increases with increasing number of received messages. However, operating at low transmit power poses the challenge that the outage probability increases the message error rate. In order to outweigh the effects of the outage probability when operating at low transmit power levels, we propose an innovative clock offset estimation algorithm that introduces the concept of *network coding* [6, p. 411] in

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the synchronization process. A particular case of a network coding strategy is the *linear network code*, in which data redundancy is added by a linear combination of the available data [7]. The advantage of linear network codes relies on their implementation simplicity. The application of NC WSN synchronization is two-fold: firstly, providing information redundancy to compensate for the inevitable message loss in the clock offset estimation process; and secondly, equalising the estimation error among the nodes in the network, as explained below.

A. Network-coded Clock Offset Estimation

Let node u_1 broadcast its clock offset measurement x_1 to its neighbours at time instant t_k, denoted as $x_1(k)$, which corresponds to synchronization round k. In general, $x_i(k) = x_i + n(k)$, where x_i is the real clock offset parameter of node u_i and $n(k)$ is the application-level additive noise with variance σ_n^2 and typically with Gaussian or exponential distribution. For example, assume that u_4 fails to receive message $x_1(k)$ whereas the remaining nodes store their reading of $x_1(k)$ in their internal memory, as shown in Fig. 1.

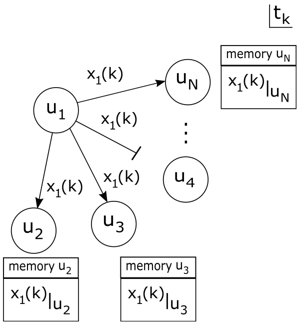

Fig. 1. Node u_1 broadcasts its clock offset measurement x_1 at time t_k.

At a following instant of time t_{k+1}, node u_2 broadcasts its own internal clock measurement $x_2(k+1)$ as well as the reading of u_1's clock offset as depicted in Fig. 2, denoted as $x_{12}(k+1)$, thus providing information redundancy with respect to $x_1(k)$. Due to the fact that this approach doubles the number of transmitted messages m per node, the transmit power S shall be halved to attain the same energy expenditure $E \propto m\,S$ [1].

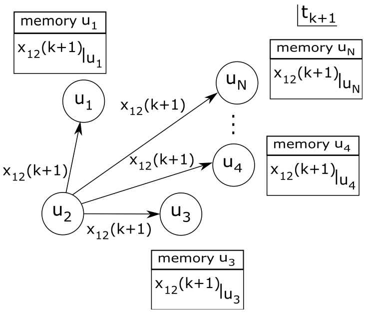

Fig. 2. Node u_2 broadcasts its measurement of node u_1's clock offset.

Noting that $x_{12}(k+1) \triangleq x_1(k+1) - x_2(k+1)$, u_4 is able to reconstruct u_1's clock offset from u_2's retransmission, denoted as $\tilde{x}_1^{(2)}(k+1)|_{u_4}$, as follows:

$$\tilde{x}_1^{(2)}(k+1)|_{u_4} = x_{12}(k+1)|_{u_4} + x_2(k+1)|_{u_4} \tag{1}$$

where $x_{12}(k+1)|_{u_4}$ is the relative clock offset between u_1 and u_2 observed from u_4, and $x_2(k+1)|u_4$ is u_2's clock measurement observed from u_4 at time t_{k+1}. The superscript in $\tilde{x}_1^{(2)}(k+1)|_{u_4}$ indicates that the estimate has been produced from a retransmission of u_2. Eq. (1) allows u_4 to reconstruct the information contained in the lost message $x_1(k)$ by combining other information available or *coded* in the *network*. In general, the reconstructed message $\tilde{x}_i^{(l)}(k)|_{u_j}$ is given by

$$\tilde{x}_i^{(l)}(k)|_{u_j} = x_{il}(k)|_{u_j} + x_l(k)|_{u_j}. \tag{2}$$

Thus, a NC estimate of u_i's clock offset produced at node u_j, denoted as $\hat{x}_i(k)|_{u_j}$, can be obtained as

$$\begin{aligned}\hat{x}_i(k)|_{u_j} \quad = \quad & \hat{x}_i(k-1)|_{u_j} + a_i \left(\hat{x}_i(k-1)|_{u_j} - x_i(k)|_{u_j}\right) + \\ & \sum_{\substack{l=1 \\ l\neq i,j}}^{N} b_l \left(\hat{x}_i(k-1)|_{u_j} - \tilde{x}_i^{(l)}(k)|_{u_j}\right)\end{aligned} \tag{3}$$

where the weighting coefficients a_i, b_l can be modified to prioritize the direct measurements $x_i(k)|_{u_j}$ over the NC reconstructions $\tilde{x}_i^{(l)}(k)|_{u_j}$, or vice versa. Both $x_i(k)|_{u_j}$ and $\tilde{x}_i^{(l)}(k)|_{u_j}$ are subject to inevitable message loss. Moreover, the computation of $\tilde{x}_i^{(l)}(k)|_{u_j}$ relies on two sets of measurements, i.e. $x_{il}(k)|_{u_j}$ and $x_l(k)|_{u_j}$, being correctly received by u_j. Therefore, let us define the error factors $\mu_i(k)|_{u_j}, \mu_l(k)|_{u_j}, \mu_{il}(k)|_{u_j}$ as follows: $\mu_i(k)|_{u_j} = 1$ if u_j receives x_i from u_i at t_k, or 0 otherwise; $\mu_l(k)|_{u_j} = 1$ if u_j receives x_l from u_l at t_k, or 0 otherwise; and, $\mu_{il}(k)|_{u_j} = 1$ if u_j receives x_i from u_l at t_k, or 0 otherwise. Then (3) can be rewritten as

$$\begin{aligned}\hat{x}_i(k)|_{u_j} = & \hat{x}_i(k-1)|_{u_j} + \alpha_i(k)|_{u_j} \left(\hat{x}_i(k-1)|_{u_j} - x_i(k)|_{u_j}\right) + \\ & \sum_{\substack{l=1 \\ l\neq i,j}}^{N} \beta_{il}(k)|_{u_j} \left(\hat{x}_i(k-1)|_{u_j} - \tilde{x}_i^{(l)}(k)|_{u_j}\right)\end{aligned} \tag{4}$$

with

$$\begin{aligned}\alpha_i(k)|_{u_j} &\triangleq a_i\, \mu_i(k)|_{u_j} \\ \beta_{il}(k)|_{u_j} &\triangleq b_l\, \mu_{il}(k)|_{u_j}\, \mu_l(k)|_{u_j}.\end{aligned} \tag{5}$$

Eq. (4) outlines the generalized NC estimate of u_i's clock offset produced by u_j at time t_k, considering the probability of error due to the channel's outage probability. By taking the expectation on both sides of (4), it is straightforward to note that $E[\hat{x}_i(k)|_{u_j}] = x_i|_{u_j}$, i.e. (4) is an unbiased estimator of x_i. Moreover, the optimal set of weighting coefficients a_i, b_l that minimizes the estimator's mean square error (MSE) can be obtained by using convex optimization methods, which will be investigated in future work.

B. Estimation Error Equalization

An extended application of NC messages in WSN synchronization is the equalization of the estimated clock offsets. For instance, consider the pairwise clock offset estimate $\hat{x}_{i,n}(k)|u_j \triangleq \hat{x}_i(k)|u_j - \hat{x}_i(k)|u_n$. Naturally, $\hat{x}_i(k)|u_j \equiv \hat{x}_{i,i}(k)|u_j$. The closed-loop sum of consecutive pairwise clock offset estimates is given by

$$\sum_{i=1}^{N} \hat{x}_{i,n}(k)|u_j = \xi(k)|_{u_j} \tag{6}$$

with $n = (i+1) \bmod N$. In a noise-free communication channel, $\xi(k)|_{u_j} = 0$. However, in real-world applications the measurement noise, disturbances and inaccuracies in the construction of the nodes lead to $\xi(k)|_{u_j} \neq 0$. Hence such error can be shared, or equalized, among all nodes in the network by updating the clock offset estimates as

$$\hat{x}_i^*(k)|_{u_j} = \hat{x}_i(k)|_{u_j} + \frac{1}{N}\,\xi(k)|_{u_j} \tag{7}$$

where $\hat{x}_i^*(k)|_{u_j}$ denotes the equalized estimate of node u_i's clock offset produced by node u_j at time instant t_k. Note that since the estimator described in (7) is also an unbiased estimator of x_i, MSE coincides with the estimator variance.

IV. HARDWARE CONSIDERATIONS IN NETWORK-CODED SYNCHRONIZATION

A typical WSN node hardware (HW) architecture consists of a low-power microcontroller unit (μC), a power unit, a transceiver, a memory unit, an analog-to-digital (ADC) converter, and a sensor device [8, p. 10], as depicted in Fig. 3.

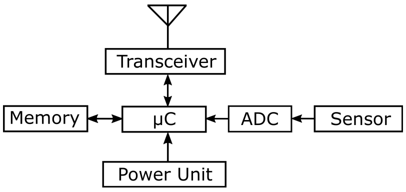

Fig. 3. WSN node architecture.

While the network-coded synchronization herein proposed does not introduce additional HW requirements to the sensor network, the following key considerations shall be made at design time:

- *Power consumption*: typically being the most important parameter in WSNs due to the limited energy resources in the nodes, the number of wireless transmissions must be carefully chosen in order to minimize the nodes' energy consumption, as detailed in [1].
- *Network size*: the algorithmic complexity of the NC synchronization is a function of the network size N. For N large, network clustering may be required in order to reduce the number of computations and transmissions per node.
- *Memory size*: careful RAM and flash memory allocation must be made due to the limited memory size in WSN nodes. The proposed algorithm has a complexity of $\mathcal{O}(N)$ on the real-valued (float data-type) clock offset estimates $\hat{x}_i(k)|_{u_j}$ and its equalized variant $\hat{x}_i^*(k)|_{u_j}$, as given by (4) and (7) respectively. This constitutes a low complexity which only grows linearly with the network size N.
- *Processing power and sampling time*: the node's microcontroller shall execute total number of code instructions within a sampling time, typically in the range of hundreds of microseconds to hundreds of milliseconds, depending on the application.
- *Data transmission rate*: the rate at which wireless messages are sent via the transceiver shall be selected based on the power consumption requirements and the desired total synchronization time. For the NC synchronization, the data transmission rates may not be necessarily in line with the microcontroller's sampling time if a message queueing mechanism is put in place in order to store the received messages.

Therefore, based on the aforementioned considerations, it can be concluded that the synchronization technique proposed in this paper is well-suite for resource-constrained WSN nodes since it does not significantly increase the processing power and memory requirements of such devices.

V. SIMULATION RESULTS

Consider a WSN containing $N = 5$ nodes labeled $u_1, u_2, \ldots, u_5$ deployed at random spatial positions, with a wireless channel experiencing Rayleigh fading effects. For example, let the real clock offsets be: $x_1 = 1.1$s, $x_2 = 1.2$s, $x_3 = 1.3$s, $x_4 = 1.4$s, and $x_5 = 1.5$s. The performance of the proposed approach is bench-marked against the well-known moving average (MA) estimator used by the Reference Broadcast Synchronization (RBS) [4, p. 20], given by

$$\hat{x}_i(k)|_{u_j} = \frac{1}{L}\sum_{r=k-L+1}^{k} x_i(r)|_{u_j} \tag{8}$$

where L is the number of samples in the MA estimation window. The MA estimation is performed at a transmit power $S_0 = 100$mW, whereas the NC estimator transmit messages are transmitted at $S_0/2 = 50$mW. The received signals are contaminated by physical-layer noise with power $\sigma^2 = 0.1$W and application-level noise with variance $\sigma_n^2 = 10^{-3}$.

Fig. 4 shows the MSE of $\hat{x}_2^*(k)|_{u_1}$ for the MA estimator and several variants of the NC estimator, as a function of the number of synchronization rounds. The latter consistently outperforms the MA estimator despite operating at half-power, by virtue of the NC information that outweighs the effects of the outage probability. Although the number of synchronization rounds dominates the MSE, varying the weighting coefficients a_i, b_l also helps reduce the MSE of the estimator, as illustrated in Fig. 5 for the Equalized NC (ENC) estimator variant.

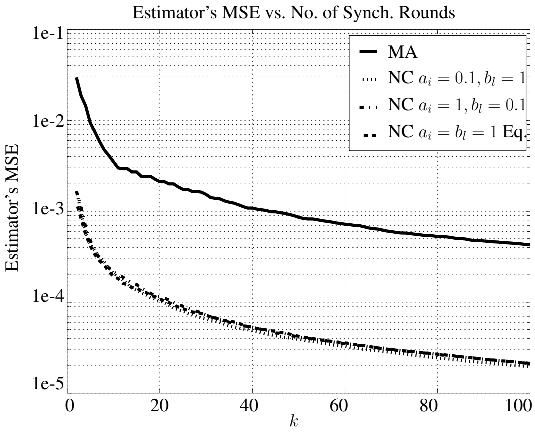

Fig. 4. MSE of the MA estimator (solid line) and MSE NC estimator with $a_i = 0.1, b_l = 1$ (dotted-line), $a_i = 1, b_l = 0.1$ (dash-dot line), $a_i = b_l = 1$ with error equalization (dash line) vs. No. of Synch. Rounds k.

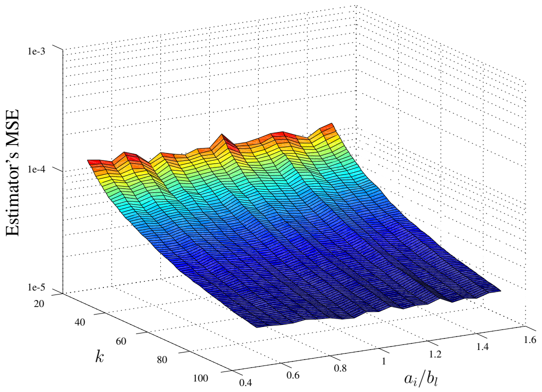

Fig. 5. MSE of the ENC estimator vs. No. of Synch. Rounds k and weighting coefficients a_i, b_l.

VI. CONCLUSIONS

The presented technique exploits network coding theory in order to outweigh the effects of the channel's outage probability effects. It has been shown that the network-coded unbiased estimator of the clock offset outperforms the well-known moving average estimator for a given energy consumption budget. Furthermore, the proposed technique is well-suited for resource-constrained WSN nodes due to its low algorithmic complexity. This constitutes a powerful tool for energy-constrained WSN's that aim to attain the expected time synchronization performance at a low energy expenditure.

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