Optimal Group Distribution based on Thermal and Psycho-Social Aspects

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ABSTRACT

In crowds, one important aspect that has been studied in literature is the sociability of groups dealing with aspects based on personality and emotions. In this paper we contribute to the space design area while considering the cultural, personality and thermal aspects to provide spatial group distribution. Our method applies a thermal comfort method together with cultural and personality model to optimally distribute the groups in a virtual environment. Results indicate that obtained groups distribution are coherent with expected based on literature.

CCS CONCEPTS

• **Computing methodologies → Modeling methodologies**; *Model verification and validation*;

KEYWORDS

virtual agents, thermal comfort, psycho-social aspects

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1 INTRODUCTION

Although many methods for crowds have been proposed since the pioneering work of crowd simulation proposed by Thalmann and Musse [15], a significantly smaller number of techniques have been proposed to deal with group behaviors. One of them was proposed by Karamouzas et al. [12], where the authors present a model in which the velocity space to plan the avoidance maneuvers of each group is used to maintain a configuration that facilitates the social interactions between the group members.

In addition to work with small groups and their internal distributions as Moussaid et al. [14], in this paper we focus on the spatial relationship among the groups, considering their cultural aspects. More precisely, we use the Hofstede method [6] to generate virtual groups with cultural characteristics observed in different Countries. Groups that are more or less open to socializing as a function of Hofstede collectivism is one example of used features. We also considered the thermal aspects in the environment and the group members personality as input to a method that tries to find the optimal location and distribution for each group in a certain environment.

This work is organized as follows: Section 2 describes some related work on groups, crowds and its connection with environments. Section 3 presents our method while Section 4 discusses obtained results.

2 RELATED WORK

This section describes some methods regarding crowd and group simulation as well as its connection and impact with the virtual environment. Section 2.1 presents some traditional methods concerning the simulation of groups and crowds, while Section 2.2 revises other methods that deal with both the simulation of the crowd and its relationship with the environment.

2.1 Group and Crowd Simulation

Through the years, many methods for crowd simulation have been proposed. One of the most traditional methods is the work of Reynolds [17], where the author obtains a realistic animation using only simple local rules. His main goal was to simulate the movement of different entities, like flock of birds, school of fishes and herd of animals. Another important work of the area was proposed by Helbing et al. [8], where the authors presented a model based on psycho-social forces to reproduce the pedestrian dynamic. One of the state-of-the-art methods to simulate crowds was proposed by Van den Berg et al. [19]. The so-called ORCA (Optimal Reciprocal Collision Avoidance) is a velocity-based method for collision avoidance between multiple agents.

Regarding group behavior, Kamphuis and Overmars [11] introduced a two-phase approach, where a path for a single agent is

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generated by any motion planner. Then, a corridor is defined around the path, where all agents stay inside. Park et al. [16] proposed to use Common Ground (CG) Theory that inherits the social realism provided by the CG model and is computationally tractable for a large number of groups and individuals. The task of navigation in a group requires effective coordination among group members.

2.2 Crowd and Design

Instead of adapting the agents of a given simulation according to the environment, there are a few works that adapt the environment itself to cope with the agents' behavior. In the work of Feng et al. [5], an approach is proposed to generate mid-scale layout designs, which are optimized in relation with crowd properties. The work of Tharindu et al. [13] also introduces a method able to generate a procedural environment according to a desired crowd behavior. Instead of altering the behavioral parameters of the crowd, the authors automatically change the environment to yield a given crowd behavior.

Cheng et al. [2] described a method where empirical thermal comfort models are incorporated in a virtual agents simulator. The authors proposed to use a simple heat transfer model to impact the environment, agent, and interpersonal heat exchange.

Our technique proposes to use thermal comfort and known methods from the literature such as Hofstede cultural dimensions [6] and OCEAN [10] (later explained) aiming to provide crowd design of environments.

3 THE PROPOSED METHOD

The core of the proposed method is to find the distribution of several groups in a spatial region that minimizes a joint discomfort measure based on psycho-social and aspects. Given the locations of the optimal group centers, we provide an approach for defining the position of each agent within the groups. Next, we present in details the discomfort measures individually, the joint measure and the procedure for group internal agents distribution.

3.1 Psycho-Social Discomfort

In his most notable work, Hofstede [6] has developed the cultural dimensions (HCD) theory. He described national cultures along six dimensions: Power Distance, Collectivism vc. Individualism, Uncertainty avoidance, Masculinity, Long Term Orientation, and Indulgence vs. restraint. We used HCD in order to characterize the social behaviors of groups, i.e. how much a group keeps close to others. In the individual level, we use the Big-five personality model (or OCEAN) [10] to describe the agents behaviors.

The input for our method is the number N of groups, the number of agents n_i and the radius r_i of each group, for $i = 1, ..., N$. We defined the radii r_i as a function of the Masculinity dimension from Hofstede (MAS), in a way that as greater is the value of MAS, the greater is the radius, as follows:

$$r_i = H_s - \left(\frac{(100 - MAS) \times 3}{100} \times \frac{(H_s - H_p)}{3} \right), \tag{1}$$

where H_s and H_p represent the radii of the social and the personal spaces defined by Hall [7], respectively. The number 3 represents the maximum cohesion value a group can achieve.

Furthermore, each group presents a "sociability" parameter $s_i \in [0, 1]$ that defines how close it wishes to stay from other groups, such that social groups are willing to stay closer to other groups, whereas anti-social groups prefer to be isolated. We map $IDV_i \in [0, 100]$ to $s_i \in [0, 1]$ through:

$$s_i = \frac{(100 - IDV_i)}{100}, \tag{2}$$

noting that larger IDV values indicate more individualistic (i.e. anti-social) groups.

For each pair of groups i and j (with $i \neq j$), we define an ideal distance D_{ij} between the two group centers $\boldsymbol{x}_i = (x_i, y_i)$ and $\boldsymbol{x}_j = (x_j, y_j)$ given by

$$D_{ij} = r_i + r_j + o_{ij}, \tag{3}$$

where

$$o_{ij} = S(2 - s_i - s_j) \tag{4}$$

is the desired "offset" between the groups, and $S > 0$ is a tunable parameter that maps the collectivity parameter to actual distances. When both groups are highly social ($s_i \approx s_j \approx 1$) we have $o_{ij} \approx 0$ and the groups tend to be closer. The opposite is expected for anti-social groups.

For each group i, its overall distance discomfort is given by

$$do_i = \sum_{j \neq i} w_{ij} \left(1 - e^{-\left| \|\boldsymbol{x}_i - \boldsymbol{x}_j\| - D_{ij} \right|} \right), \tag{5}$$

which increase as the center distances $\|\boldsymbol{x}_i - \boldsymbol{x}_j\|$ gets farther from the ideal distance D_{ij}. Parameter w_{ij} is a weight that tries to enforce the ideal distances for pairs of groups that present strong sociability, given by

$$w_{ij} = \frac{e^{-\beta o_{ij}}}{\sum_j e^{-\beta o_{ij}}}, \tag{6}$$

where β controls the sociability decay. If β is large, w_{ij} will rapidly decay to zero if either s_i or s_j are small, so that the optimal distance would only be enforced to pairs of highly social groups. In this work we experimentally selected $\beta = 1$.

In Eq. (3), groups that present a small sociability s_i lead to smaller weights w_{ij}, meaning that enforcing the optimal distance is not important. In fact, anti-social groups wish to stay farther away from any other group, and to enforce this behavior we also define a second penalization term

$$dm_i = \max_{j \neq i} (1 - w_{ij}) e^{\gamma (\|\boldsymbol{x}_i - \boldsymbol{x}_j\| - D_{ij})}, \tag{7}$$

so that pairs of groups with low sociability (i.e. small w_{ij}) will tend to be farther away than the pairwise optimal distance, not closer.

Finally, a third penalization term is created to strongly discourage intersection of groups. If I_{ij} defines the Intersection over Union (IOU) between the circles that represent groups i and j, we would like to penalize any non-zero value for I_{ij}. Let us recall that the IoU is the area of the intersection divided by the area of the union of both sets, being restricted to the range $[0, 1]$ The chosen penalty term is given by

$$di_i = M \max_{j \neq i} I_{ij}^2, \tag{8}$$

where M is a large positive constant to produce large penalty values even for small IoU values (we defined $M = 10,000$ in our experiments), to avoid groups interpenetrations.

The final psycho-social discomfort term for group i is given by

$$ds_i = 100\left(\delta di_i + (1-\delta)\left(\frac{do_i}{2} + \frac{dm_i}{2}\right)\right), \tag{9}$$

where δ controls the importance of the interpenetration term (we used $\delta = 0.1$).

3.2 Thermal Discomfort

Thermal comfort/discomfort has been studied for years by environmental engineers [3], and there are six primary factors that impact thermal comfort. Two of them (metabolic rate and clothing insulation) depend on the individual, and the remaining four (air temperature, mean radiant temperature, air speed and humidity) are environmental. There are several studies that try to relate these parameters with the human perception of thermal comfort, and the **Predicted Mean Vote** (PMV) model [3] maps the six key factors for thermal comfort onto a numerical thermal sensation scale ranging from -3 (cold) to +3 (hot), 0 being the neutral (ideal) feeling.

Based on the PMV, Fanger [3] also tried to estimate the **Predicted Percentage of Dissatisfied** (PPD), which is the expected percentage of people thermally dissatisfied in a given environment, leading to the following relation:

$$PPD = 100 - 95e^{-0.03353PMV^4 - 0.2179PMV^2}. \tag{10}$$

Although the PPD indicates the percentage of people dissatisfied in a given environment, it can also be viewed as the probability of an individual agent feeling uncomfortable in the same environment, as recently explored in [2].

In this work, the thermal discomfort map f is any smooth function $f : \Omega \rightarrow [0, 100]$ that maps the thermal discomfort of an agent at any location of the spatial domain Ω under analysis. If heat sources/sinks in the environment are known, a realistic thermal map can be obtained by applying the heat diffusion equation and computing the PMV/PPD, as done in [2].

Since our method deals with groups and not individuals, we must define a thermal discomfort level for each group. This level can be the average discomfort of the group, or the maximum discomfort level within the region that represents the group. In this work, we chose the latter approach, so that a group feels comfortable only if *all* members in the group are comfortable.

Since finding the maximum discomfort of function f in a circular region is not a trivial task (particularly if f is defined numerically), we approximate the maximum value by sampling f in five cross-shaped points within the circular region. If $\boldsymbol{x}_i = (x_i, y_i)$ is the group center and r_i is its radius, the discomfort of group i is given by

$$\begin{aligned} dt_i = \max\{&f(x_i, y_i), f(x_i + r_i, y_i), f(x_i - r_i, y_i), \\ &f(x_i, y_i + r_i), f(x_i, y_i - r_i)\}. \end{aligned} \tag{11}$$

3.3 Joint Discomfort Term

The final configuration of the groups in the spatial region should minimize a joint discomfort term that combines psycho-social (ds) and thermal (dt) aspects. A precise formulation of this term is a challenging task, due to complex individual/cultural aspects. For instance, members of a group might be more tolerant to uncomfortable temperatures, but are not fond of high-density crowds.

In this work, the final objective function used in the optimization is the sum (for all groups) of a weighted linear combination of psycho-social and thermal discomfort terms, given by

$$d = \sum_{i=1}^{N} \alpha dt_i + (1-\alpha) ds_i, \tag{12}$$

where $\alpha \in [0, 1]$ controls the balance between the two terms. Note that d is a function of the group centers $\boldsymbol{x}_1, \boldsymbol{x}_2, ..., \boldsymbol{x}_N$, which need to be optimized.

To solve the optimization problem, we explore an interior-point approach [1] implemented by MATLAB's `fmincon` function. To cope with different results caused by different initializations, we run each experiment 10 times and select the one with the smallest value for the objective function d.

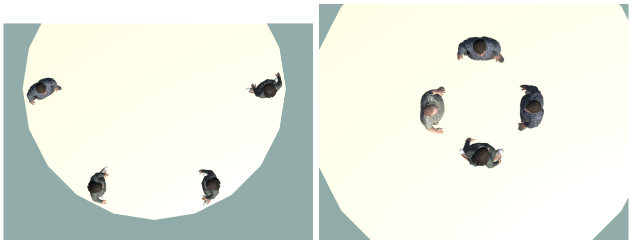
3.4 Group Internal Agents Distribution

Once we have the result of the optimization process generating the center of each group, we are able to distribute the agents into the group's region. As mentioned before we use OCEAN model also known as the Big Five [10], which main characteristics are: Openness to experience ("the active seeking and appreciation of new experiences"); Conscientiousness ("degree of organization, persistence, control and motivation in goal-directed behavior"); Extraversion ("quantity and intensity of energy directed outwards in the social world"); Agreeableness ("the kinds of interaction an individual prefers from compassion to tough-mindedness"); Neuroticism ("how much prone to psychological distress the individual is").

Based on the OCEAN description we decided to use the Openness and Extraversion dimensions since they have more impact on the social behaviors [4]. The Extraversion E_i of agent i was used as an attraction factor to the center of the group. The Openness O_i was simply used to describe how much agent i is open to integrate the group conversation, so we mapped that to the agent direction vector. The idea behind our choice is that when O_i is high, agent i is open to talking to everybody in the group, so it looks to the center. When an agent has a low value for O it states for an agent focused on one of the group elements, so looking direct to him/her. We included a little noise (-10%, $+10\%$) in the O and E values to avoid very discretized behaviors. Figure 1(a) shows an example of the internal distribution of agents in a group when their O are low and Figure 1(b) illustrates when their O values are high.

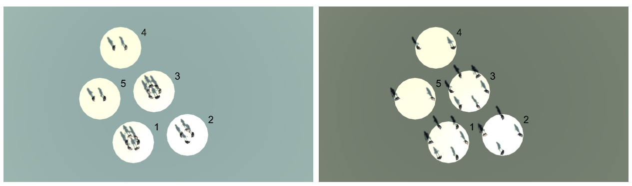
4 EXPERIMENTAL RESULTS

This section presents some experimental results. First, we defined six base scenarios using five different Countries: Brazil, India, Germany, Chile and Japan. Although we can interactively control the value of parameters: MAS, IDV (from HCD) and O and E (from OCEAN), we decided to use the available values of specific countries, as presented in the literature. Each one of first five scenarios has the same number of groups (5), number of agents per group (6,4,6,2,2) and are parametrized according to data from one of the specified Countries, in order to produce scenarios that can be comparable. The last scenario also contains five groups, in this case having the same number of agents (i.e. 4), but each group characterized according to a different Country. For each Country we



(a) Agents with LOW values for O and E.
(b) Agents with HIGH values for O and E.

Figure 1: Difference on the positioning and orientation of the agents of a group, for both low values (a) and high values of O and E.



(a) High values for E
(b) Low values for E

Figure 2: Two images representing the impact of E in the agents distribution into the groups.

considered the values of HCD and OCEAN, available on [9] and [18], respectively, as defined in Table 1.

Table 1: Values for Hofstede (MAS and IDV) and OCEAN (O, and E) for each chosen Country (Brazil, Chile, Japan, India and Germany, respectively). Hofstede's values were extracted from the Hofstede Insights website [9], while OCEAN's values were extracted from [18].

	BRL	CHL	JPN	IND	GER
MAS	49	28	95	56	66
IDV	44	31	88	51	83
O	49.1	54.7	41.5	48.5	47.8
E	45.9	47.5	46.7	47.4	50.3

As a result, we can observe two emergent effects that depend on Psycho-social comfort. Firstly, groups can be impacted by the psycho-social comfort, being close/far to other groups depending on their cultural aspects, in particular, their IDV (individualism) [6]. Also, the radius of each group is impacted by the MAS from Hofstede [6] too. Secondly, when groups are located in the environment, agents distribution into the groups are dependent on their OCEAN parameters [18], so agents can be more close to the center of the group (depending on E value) and also agent direction vector can be focused on one other agent or to the hole group (depending on the O parameter).

Figure 3 shows the groups distribution in the environment dependent on HCD and OCEAN from 5 Countries without considering the thermal discomfort (i.e., using $\alpha = 0$ in Eq. (12)). It is easy to see that the country with the highest value for MAS is Japan, since they are a less collective population, while Chile presents the opposite behavior according to the literature. We also computed the average distance among the groups from same scenario to show the differences in a quantitative way, as presented in Table 2, and such data confirmed that Japan and Chile are extreme opposite in the input data and also in our model.

Table 2: Average distances among group centers for each scenario.

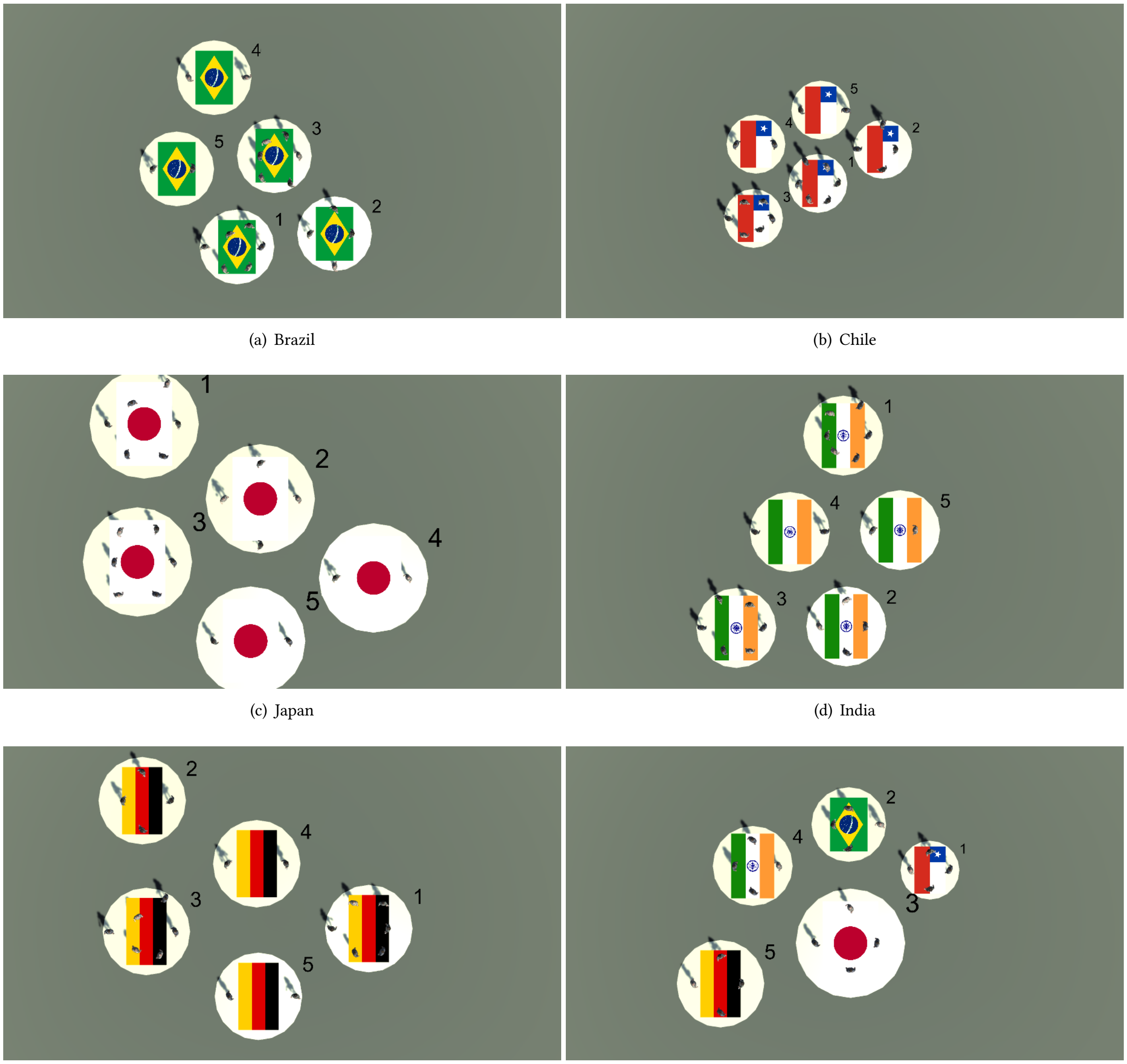
Scenario ID	AVG distance
Brazil	7.817553
Chile	5.823312
Germany	10.33314
India	8.740937
Japan	10.99607
Mixed	9.092193

In addition, we simulated fictitious groups (not coming from Countries parameters) to show more clearly the effect of the parameters, in particular s_i. More precisely, we simulated four scenarios with the same number of groups and agents of the previous experiments, and each group with the same radius. The only parameters that have been changed in these examples were the IDV, which impact s_i: in the first example, all groups have low IDV values; in the second, only one group (group 2) presents high IDV value; in the third, groups 2 and 5 present high IDV values; and in the last experiment, all groups present high IDV values. The final positioning of the groups, without thermal discomfort, is shown in Figure 4. As it can be observed, groups with high collectivity (low individualism) stay closer together, and groups with high IDV values stay more isolated.

In a final experiment, we reproduced the setting using in Figure 4 but adding a heat source in the middle of the virtual scenario (modeled as an isotropic 2D Gaussian thermal discomfort function f centered at the origin), and used equal weights for both thermal and psycho-social discomfort terms (i.e., $\alpha = 0.5$ in Eq. (12)). The results are shown in Figure 5, and the thermal discomfort is shown in a `jet` colormap, so that high discomfort levels are shown in red, and low values in dark blue. As can be observed, the groups respected their distance to other groups based on the IDV level, and at the same time avoided the thermally uncomfortable region in the middle of the region. In particular, it is interesting to compare the results with the thermal discomfort term and their counterparts without this term, shown in Figure 4.

5 FINAL CONSIDERATIONS

This work presented a method to simulate crowds taking into account thermal comfort and psycho-social aspects of groups. This information was used to determine the best position for groups of agents, as well as the positioning and behavior of the individuals within each group. The results achieved by our method were coherent with the expected theoretical result: individual groups (i.e., high IDV_i and small s_i) tend to keep isolated from the other groups.



(a) Brazil (b) Chile

(c) Japan (d) India

(e) Germany (f) All countries

Figure 3: Distribution of groups and agents for all chosen countries (Brazil (a), Chile (b), Japan (c), India (d) and Germany (e)), as well for the scenario with all countries together (f).

When the thermal discomfort term is used, all the groups tend to keep their relative distances based on their IDV values, and at the same time avoid thermally uncomfortable regions. We have also used psycho-social data related to actual countries, and the group distributions was coherent with what one would be expected in real life, based on the literature.

There are several avenues left for future work. At the moment, the weight α that combines thermal and psycho-social terms is the same for all groups, but this parameter can be tuned individually for each group by using one α_i blend for each group i in Eq. (12). Another possibility would be to consider individual thermal discomfort maps for hot and cold scenarios, as provided by the PMV introduced by Fanger [3]. Such choice can be motivated by the fact that some cultures are more tolerant to cold (e.g., Nordic) than hot, while others (such as Equatorial) present an opposite thermal bias.

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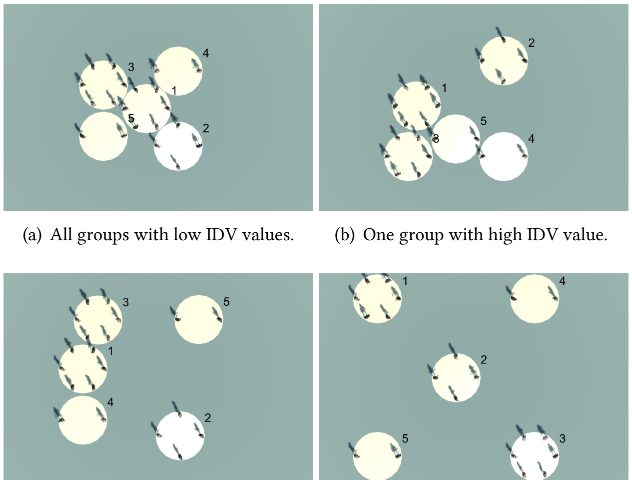
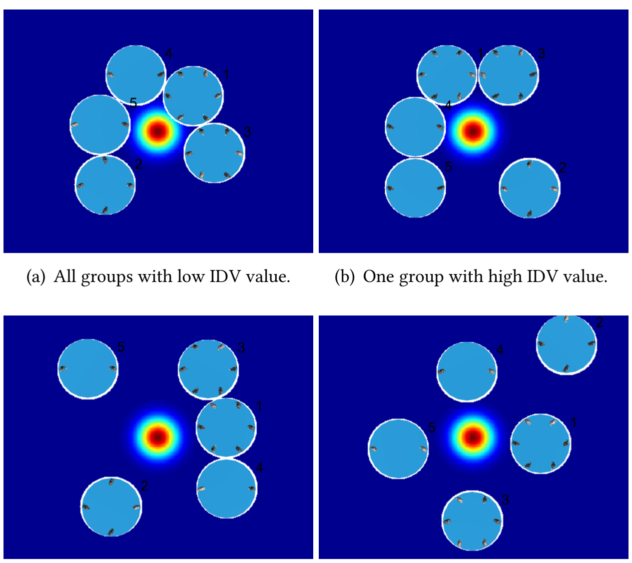


(a) All groups with low IDV values. (b) One group with high IDV value.

(c) Two groups with high IDV value. (d) All groups with high IDV value.

Figure 4: Distribution of groups in four fictitious experiments containing the same value for radius, however varying the number of collective groups. No thermal discomfort was used.



(a) All groups with low IDV value. (b) One group with high IDV value.

(c) Two groups with high IDV value. (d) All groups with high IDV value.

Figure 5: Distribution of groups in 4 fictitious experiments containing same value for radius, however varying the number of collective groups. Thermal discomfort added with weight $\alpha = 0.5$, and thermal discomfort shown as a jet colormap (higher values in red, smaller in dark blue).

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