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Structured Markovian models for discrete spatial mobile node distribution

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Abstract The study and characterization of node mobility in wireless networks is extremely important to foresee the node distribution in the network, enabling the creation of suitable models, and thus a more accurate prediction of performance and dependability levels.

In this paper we adopt a structured Markovian formalism, namely SAN (Stochastic Automata Networks), to model and analyze two popular mobility models for wireless networks: the Random Waypoint and Random Direction.

Our modeling considers mobility over a discrete space, i.e., over a space divided in a given number of slots, allowing a suitable analytical representation of structured regions. We represent several important aspects of mobility models, such as varying speed and pause times, and several border behaviors that may take place. One, two, and three-dimensional models are presented. For the two-dimensional models, we show that any regular or irregular convex polygon can be modeled, and we describe several routing strategies in two dimensions.

In all cases, the spatial node distribution obtained from the steady state analysis is presented and whenever analogous results over continuous spaces were available in the literature, the comparison with the ones obtained in this paper is shown to be coherent.

Besides showing the suitability of SAN to model this kind of reality, the paper also contributes to new findings for the modeled mobility models over a noncontinuous space.

Keywords Structured stochastic modeling formalisms · Mobile nodes · Markovian models

1 Introduction

Mobility keeps gaining in importance, playing a significant role in the design and implementation of communications infrastructures and distributed systems. The study of mobility in a broad spectrum has impacted a wide range of technical results such as new process calculi [40], new distributed programming paradigms [23], and a large set of new and revisited distributed algorithms and protocols, such as reported in [1, 4, 39, 45] to mention a few.

Methods for the representation and analysis of distributed systems are important since such systems are often too expensive to prototype and nontrivial to monitor aspects that become more challenging when mobility is involved; especially, the quantitative analysis of mobile systems becomes of paramount importance. Very often the properties of mobile systems can only be considered in a quantitative way due to the several aspects involved and wide possibility of behavior. The study and characterization of mobility models is very important in order to analyze a given distributed system where mobility is present in one of the possible forms.

The analysis of mobility models found in the literature proposes simulation or analytical solutions. While typical analytical solutions under use may provide more reliable indices than simulation, the representation of some aspects of a given system may become very complex or lead to intractable models.

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One of the cases rarely considered is the spatial distribution when the nodes move over a discrete space. Such situation may occur when nodes move on a grid like a net of corridors in a building or streets in a city. An even clear situation of discrete space are coarser models, where the exact position is not needed, but the identification of a room or access point in use is the relevant information.

It is not a coincidence that such cases of mobility are not usually modeled, since either their mathematical formulation becomes quite complex or state based models for these realities easily become intractable due to the huge number of possible states, i.e., the well-known state space explosion problem. In fact, other initiatives to model similar realities, e.g., street movement [14, 48] does not tackle discrete space, but rather consider a set of continuous lines.

On the other side, structured formalisms such as Stochastic Petri Nets [2], Stochastic Process Algebra [27], and Stochastic Automata Networks—SAN [44] are inherently discrete state and provide powerful concepts to stepwise build detailed mobility models. Probably the greatest advantage of using structured formalisms is the possibility to perform numerical solutions rather than simulation experiments. Numerical solutions deliver exact probabilities that does not suffer from the accuracy and statistical relevance issues common to simulation approaches [17].

Actually, the numerical analysis literature [12, 18, 19, 33] acknowledges the computational benefits to use structured representations in analytical modeling and numerical solution. Therefore, we consider that the discussion of the suitability of such methods to describe and solve mobility models is important. Among the options of structured formalisms, we have chosen SAN basically due to the authors previous experience. To our experience, the use of other above mentioned formalisms would lead to analogous discussions and results.

In this paper, we focus on the analysis of mobility models typically employed in wireless network, more specifically we model and analyze two mobility models: Random Waypoint and Random Direction. The Random Waypoint mobility model [5] is one of the most used in wireless networks research [7, 8, 11, 28, 32, 38]. The Random Direction [47] was also chosen since it is considered an approximation of real movement behavior for several classes of applications and, therefore, often used [5, 21, 26, 43, 51]. The main objective of this analysis is to derive the spatial node distribution which affects directly the performance and dependability of the used protocols [34, 43]. In this context, our contribution is the following:

- We introduce the use of SAN formalism and show that this method lends itself to the formalization of mobility models, providing explicit and non-ambiguous description of such models. Besides, the structured nature intrinsic of the SAN formalism allows to stepwise enrich the description level of detail when compared to continuous-state models.
- We formally describe the Random Waypoint mobility model in SAN and show the compatibility of our results with existing continuous state space studies for square areas. Further, we extend the analytical model of the Random Waypoint to analyze the impact of representing more detailed aspects, namely a *pause* time while the mobile nodes do not move and the choice of different *routing strategies*. Note that such analysis of routing strategies is only meaningful due to the use of a discrete state space modeling adopted.
- We show the ability to model any surface in the shape of regular or irregular convex polygons. The modeling of such surfaces was carried out considering the Random Waypoint mobility model. Additionally, the resulting spatial node distributions for surfaces in the shape of some regular polygons, such as hexagon and triangle, are coherent with those found in the literature considering continuous spatial distribution.
- We formally describe the Random Direction mobility model using SAN and we show the results achieved with the spatial node distribution. For instance, we show that the pause does not affect the spatial node distribution probability in the one and two dimensional cases. An analysis of nonsquare surfaces is not represented in this paper, but it can be developed analogously to the presented for Random Waypoint models.

It is important to point out that it is not the goal of this paper to defend the use of Random Waypoint and Random Direction as valid models for describing human mobility, or any other particular behavior. For those issues, the interested reader may find extensive and relevant material in [24, 30, 37]. The choice of modeling RWP and RD is based on the abundance of discussion and results for these mobility models in the area.

This paper is structured as follows: Sect. 2 describes Random Waypoint and Random Direction mobility models and surveys the related works; Sect. 3 describes the Stochastic Automata Network formalism; Sect. 4 proposes the SAN models and analyzes the results for the Random Waypoint model in one and two dimensions, for square and other (regular and irregular) convex polygonal surfaces; Sect. 5 introduces one- and two-dimensional SAN descriptions for the Random Direction mobility model, discussing possible variations of these SAN models and presenting the results achieved in terms of spatial node distribution. Final remarks and some more elaborated modeling efforts (3D models) are presented in the conclusion with some possible future works.

2 Mobility models

The literature presents many studies classifying different mobility models for wireless networks [5, 13, 31]. These models are used to represent the movement of mobile nodes in a wireless network, and they can be split in individual and group models. The individual mobility models represent the independent movement behavior of one mobile node, while the group mobility models are used when a group of nodes have a common behavior in the choice of their movements. The modeling of individual mobility is simpler to implement, because the node movement can be done through independent events. Due to this characteristic, the individual models are the most used to evaluate wireless networks, as in [3, 6, 7, 31]. The quantitative analysis of such models directly influences the estimation of performance and dependability levels of wireless networks.

As mentioned in the previous section, in this work, we focus on the individual mobility models Random Waypoint and Random Direction, which are discussed below.

2.1 Random waypoint mobility model

In the Random Waypoint Mobility model [7, 13], the mobile node randomly chooses a destination and moves to this destination with constant speed. When reaching the destination, the mobile node remains stopped for some time and then begins the whole process all over again with the same rules. This model and its variations are widely used. Some examples of its use can be found in [7, 8, 11, 28, 29, 32, 38, 42, 46].

Several authors consider the Random Waypoint to obtain the spatial node distribution, allowing one to derive important performance and dependability parameters. Studies about node connectivity using Random Waypoint as mobility model can be found in [15, 29, 36, 49].

Royer et al. [47] carry out an analysis by simulation on the spatial node distribution for the Random Waypoint. In [47], it was noticed a higher probability of the node being in the center of the simulated area. A detailed analytical study of spatial node distribution generated by the Random Waypoint mobility model was presented by Bettstetter in [7]. The results achieved there were also validated by simulation studies for one- (line) and two-dimensional (square) models without pause time.

Resta and Santi in [46] generalize the analysis of [7] allowing pause time and speed to be changed. They derive an explicit formula of the one-dimensional spatial node distribution, and an approximated formula for the two-dimensional case.

Hyytiä et al.in [29] derive an explicit expression for the spatial node distribution of Random Waypoint in convex domains and they also demonstrate the use of the result for various shapes of the domain. They derive an polynomial approximation for density function for regular triangles, squares, and hexagons, comparing some of their results with [7]. Besides, they analyze Random Waypoint models and some applications, such as connectivity and traffic load in ad hoc networks.

The studies have shown that although the initial node positioning is taken from an uniform random distribution, the mobility model changes this distribution during the movement. This effect occurs because nodes tend to cross the center of the modeled region with a relatively high frequency and, without a pause time, the choices are independent of the node speed. However, as the pause time increases, the node distribution approximates the uniform distribution.

The Random Waypoint mobility model is also used in others contexts. Jayakumar et al. [32] and Kumar et al. [35] used Random Waypoint to evaluate routing protocols to MANET (Mobile Ad hoc Network). Besides, a study in the context of cellular networks is presented by Hyytiä et al. in [28].

2.2 Random direction mobility model

The Random Direction is another mobility model used to analyze and simulate the movement of mobile nodes in a wireless network. Different from the Random Waypoint, the choice of new destination is done through a new direction, a speed to travel, and a time duration for this travel [25, 41].

One important aspect of the Random Direction mobility model is the possible behavior when reaching the border of the area considered, i.e., the border behavior. In [5], Bettstetter shows three different border rules: *Bounce, Delete and Replace,* and *Wrap Around*. These border rules guarantee that the number of nodes remains constant during the analysis. In the *Bounce* border rule, when the node achieves the border, it reflects back to the simulation area. The new angle is chosen according to the previous angle with the reached border [26, 47]. In the *Delete and Replace*, according to [5], the node is deleted and a new node is created on a randomly chosen point in the simulation area. In the *Wrap Around*, the node continues the movement and reenters in the opposite border with the same speed and direction parameters. This approach models a torus shape surface [22].

There are some variations of Random Direction mobility model in the literature. In [26], Haas and Pearlman present a mobility model considered as a simplified version of Random Direction Mobility model described in [5]. In this model, the mobile node selects a speed and a direction, between 0 and 359 degrees, and moves in this direction until the border is reached. Once the border is reached, the mobile node chooses another direction between 0 and 180 degrees and continues the process.

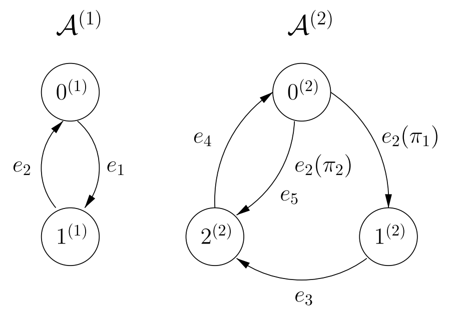
In the basic model considered by Royer et al. [47], the mobile node randomly selects a direction and a destination along this traveling direction, thus it is not forced to travel until the border. Besides, a speed is also randomly chosen. Once it reaches the destination, it remains stationary for some predefined pause time. At the end of the pause time, the process is repeated. If a border of the simulation area is reached, it reflects back into simulation area, using the *Bounce* border rule. Some results with simulations about traffic load and node density using this model are presented in [47] and [43].

In [5], with the aim of providing a better of mobility behavior to real mobility behavior, Bettstetter employs a combination of principles for direction and speed control that makes the movement of users smoother. A new destination is chosen by choosing a new direction. The speed and direction changes are both probabilistic. Two stochastic processes are used: one process determines at what time a mobile station changes its speed, and the other process determines when the direction is changed. The speed is changed incrementally by the current acceleration of the mobile node, and also the direction change is smooth: the direction is changed in several steps until the new target direction is achieved. Besides, the author analyzes the spatial node distribution using Random Direction as mobility model and modifying the rate in which the speed and the direction changes. The border behavior used was *Delete and Replace*, since the others presented an uniform spatial node distribution. Our approach in Sect. 5 is similar to the principles adopted in [5].

Further work on the Random Direction model is also presented in Di et al. [21], where a mathematical model of a 3D space is proposed and compared to simulation results. Analogously to the Random Waypoint, the Random Direction model is used in he evaluation of MANET routing protocols such as Ad-hoc On-Demand Distance Vector Routing (AODV), Destination Sequenced Distance Vector (DSDV), Dynamic Source Routing (DSR) and Temporally-Ordered Routing Algorithm (TORA) [35].

3 SAN—stochastic automata networks

The SAN formalism was proposed by Plateau [44] and its basic idea is to represent a whole system by a collection of subsystems with an independent behavior (*local transitions*) and occasional interdependencies (*functional rates* and *synchronizing events*). Each subsystem is described as a stochastic automaton, i.e., an automaton in which the transitions are labeled with probabilistic and timing information. Hence, one can build a continuous-time stochastic process related to a SAN model, i.e., a SAN model has an equivalent Markov chain model [9, 50].



$$f = \left[\left(st\ \mathcal{A}^{(2)} == 0^{(2)}\right) * \lambda\right] + \left[\left(st\ \mathcal{A}^{(2)} == 2^{(2)}\right) * \gamma\right]$$

Type	Event	Rate
loc	e_1	f
syn	e_2	μ
loc	e_3	σ
loc	e_4	δ
loc	e_5	τ

Fig. 1 Example of a SAN model

There are two types of events that change the state of a SAN model: *local events* and *synchronizing events*. Local events change the model state changing the state of only one automaton. Synchronizing events, in opposition, can change simultaneously the states of more than one automaton.

The other possibility of interaction among automata is the use of functional rates. Any event occurrence rate may be expressed by a constant value in $\mathbb{R}^+$ (a positive real number) or by a function of the state of other automata into $\mathbb{R}^+ \cup \{0\}$. In opposition to synchronizing events, functional rates are one-way interaction among automata, since it affects only the automaton where it appears and not the automata from which it depends. Figure 1 presents a SAN model with two automata, one synchronizing (e_2) and four local events (e_1, e_3, e_4 and e_5).

In this example, the rate of the event e_1 is not a constant rate, but a functional rate f described with the SAN notation[1] employed by the PEPS software tool [10]. The interpretation of f defines the firing of the transition from state $0^{(1)}$ to $1^{(1)}$ with rate λ if automaton $\mathcal{A}^{(2)}$ is in state $0^{(2)}$, or rate γ if automaton $\mathcal{A}^{(2)}$ is in state $2^{(2)}$. If automaton $\mathcal{A}^{(2)}$ is in state $1^{(2)}$, the transition from state $0^{(1)}$ to $1^{(1)}$ does not occur (rate equal to zero). It is important to observe that the use of functions allows a compact and flexible way to describe in one single (local or synchronizing) event alternative behaviors [9].

Another important point to observe in this example is the event e_2, where two alternative occurrences exists for au-

[1] The interpretation of a function can be viewed as the evaluation of an expression of non-typed programming languages, e.g., C language. Each comparison is evaluated to value 1 (*true*) and value 0 (*false*).

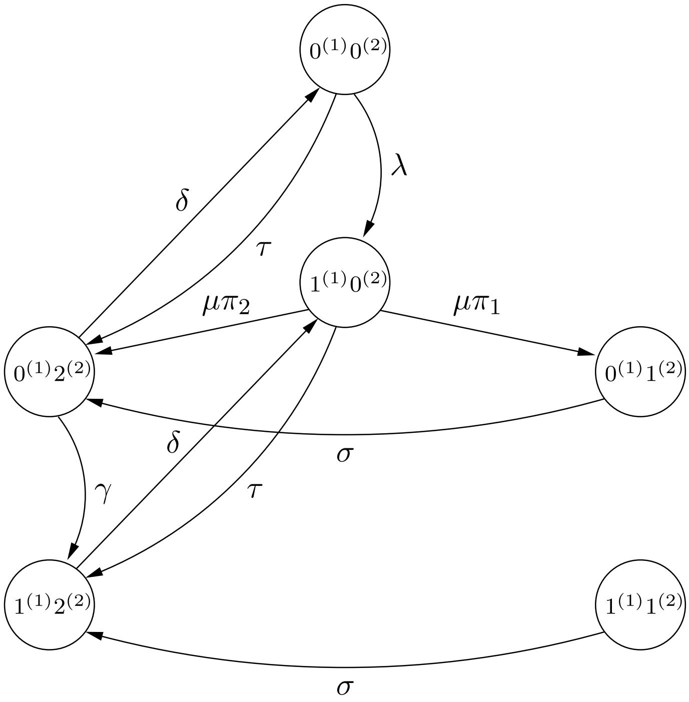


Fig. 2 Equivalent Markov chain

tomaton $\mathcal{A}^{(2)}$. The occurrence of event e_2 could then change the state of automaton $\mathcal{A}^{(1)}$ from $1^{(1)}$ to $0^{(1)}$ at the same time when the automaton $\mathcal{A}^{(2)}$ changes from state $0^{(2)}$ to alternatively $1^{(2)}$ (with probability π_1) or to $2^{(2)}$ (with probability π_2). Actually, every time a local or synchronizing event may change a local state to two or more other states a probability of choice must be defined.

Figure 2 shows the equivalent Markov chain model to the presented SAN model. Assuming the state $0^{(1)}0^{(2)}$ as an initial state, only five of the six states in this model are reachable. To deal with such models, it is usual to express either: an initial state and let the reachable state space be computed by possible firing sequences; or a function within the product state space returning a nonzero value for reachable global states.

For the model in Fig. 1, the reachability function must exclude the global state $1^{(1)}1^{(2)}$, thus:

$$\mathit{Reachability} = ! \big[\big(st\,\mathcal{A}^{(1)} == 1^{(1)} \big) \&\& \big(st\,\mathcal{A}^{(2)} == 1^{(2)} \big) \big]$$

Roughly, we can say that a SAN model complexity to compute a stationary or transient solution is usually proportional to the size of its product state space, i.e., the number of possible combinations of local (automata) states. However, according to the solution method chosen, this can vary considerably. Nevertheless, from an efficiency general perspective it is almost always interesting to obtain a SAN model with minimum product state space, i.e., a model with the smaller number of automata, each one of them with as few local states as possible.

4 Random waypoint

Within this section, we discuss one and two-dimensional SAN models for the Random Waypoint mobility. Part of these results were presented in [20] where only linear (1D) and square (2D) areas were considered. In this section, we show these previous results and extend to consider 2D areas that can be any convex (even irregular) polygons.

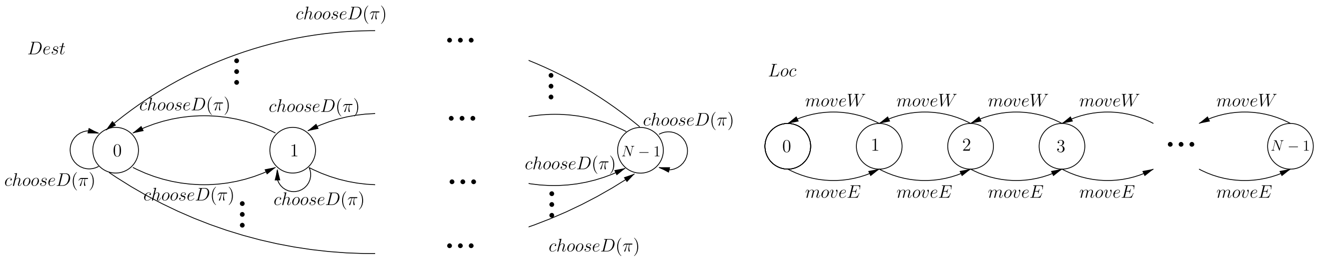
4.1 Random waypoint 1D SAN model

In the one-dimensional case, we define a SAN model to represent a region of M meters modeled as a line with N slots, where one node can move to the east or to the west. Model such reality using a discrete space formalism is quite intuitive. The observations fulfilled the expectations in the sense that the greater the N, the more the results are similar to those found in the literature for continuous space situations.

Figure 3 describes a SAN model composed of two automata, called *Dest* and *Loc*. Automaton *Dest* represents the randomly chosen destination. Automaton *Loc* represents the node's actual location. Both automata have N states, and each state represents a slot with size M/N meters.

There are two basic general situations for such SAN model. Either the node is *moving* to a chosen destination ($stDest \neq stLoc$), or the node is in the chosen destination ($stDest = stLoc$) waiting (*pause*) for the choice of another direction.

Automaton *Dest* has only one event called *chooseD* (choose a destination) which may only occur when the



Type	Event	Rate
loc	$moveW$	f_W
loc	$moveE$	f_E
loc	$chooseD$	f_D

$f_W = rtSpeed * (stDest < stLoc)$
$f_E = rtSpeed * (stDest > stLoc)$
$f_D = 1/Pause * ((stDest) == (stLoc))$
$\pi = 1/N$

Fig. 3 Random waypoint 1D SAN model

model is in the *pause* situation. Such situation is represented by both automata *Dest* and *Loc* being in the same state. The time to stay in this *pause* situation (*Pause*) is represented by the inverse rate of event *chooseD* ($1/Pause$), which is conditioned by the function representing both automata in the same state. The resulting rate of event *chooseD* is then expressed by the function f_D in Fig. 3 SAN model:

$$f_D = \frac{1}{Pause} * \big((stDest) == (stLoc)\big)$$

This function returns $1/Pause$ when the model is in the *pause* situation, or zero if the model is the *moving* situation. However, to represent the choice of all possible destinations there must be transitions between every pair of states of automaton *Dest*, i.e., from any state in *Dest* there will be N possible transitions with event *chooseD*. To correctly represent that aspect in the SAN model, each occurrence of event *chooseD* must be tempered with a probability π. As mentioned in Sect. 3, when a same event has many possible transitions from a same state, probabilities summing 1 should be assigned to all possible events. Assuming an equally distributed choice of direction, probability π should be equal to all transitions, i.e.: $\pi = \frac{1}{N}$.

Automaton *Loc* has two possible events representing the node's movement toward West (*moveW*) or East (*moveE*) directions. Events *moveW* and *moveE* can only occur when the model is in the *moving* situation, i.e., when automata *Dest* and *Loc* are not in the same state. Since this model describes an one-dimensional case, once again there are just two possibilities: either the destination position is at the west side on the current node location ($stDest < Loc$), or it is at the east side ($stDest > Loc$). The moving speed itself is modeled by the rate of these events (*rtSpeed*) as the inverse of the time the node stays at each slot. This residence time is simply the reason between the node average speed in meters per time unit (*speed*) times the size of the slot region in meters (M/N). Hence the rate of the events *moveW* and *moveE* are respectively:

$$f_W = rtSpeed * \big((stDest) < (stLoc)\big)$$

and

$$f_E = rtSpeed * \big((stDest) > (stLoc)\big)$$

where $rtSpeed = \frac{Speed}{\frac{M}{N}}$.

4.1.1 Validating the 1D model

To validate the proposed model, we refer to the results obtained by Bettstetter, Resta, and Santi for the Random Waypoint behavior [7]. In this work, a general one-dimensional case with no pause time ($pause = 0$) has the spatial node probability distribution defined by the following probability density function:

$$f(x) = -\frac{6}{a^3}x^2 + \frac{6}{a^2}x \tag{1}$$

where $a > 0$ is the size of the observed region and x, such that $0 < x < a$, is the position in this area. The main important observation for this formula is that it represents the distribution in the region as a continuous function. In order to compare our model results with [7], we need to assume a number of discrete space slots. It is important to notice that in (1) the node speed is not relevant, which makes sense for the zero pause case, since with no pause time the speed does not affect the spatial node distribution. However, our model does not ignore neither the pause time, nor the node speed.

We also compared our results with the results presented in the further work of Resta and Santi [46], which defines the expression in (2) to represent the probability distribution considering pause time (*Pause*) and node speed (v).

$$\begin{aligned} f(x) = {} & p_{stat} + (1 - p_{stat})p \\ & + (1 - p_{stat})(1 - p)6x(1 - x), \end{aligned}$$

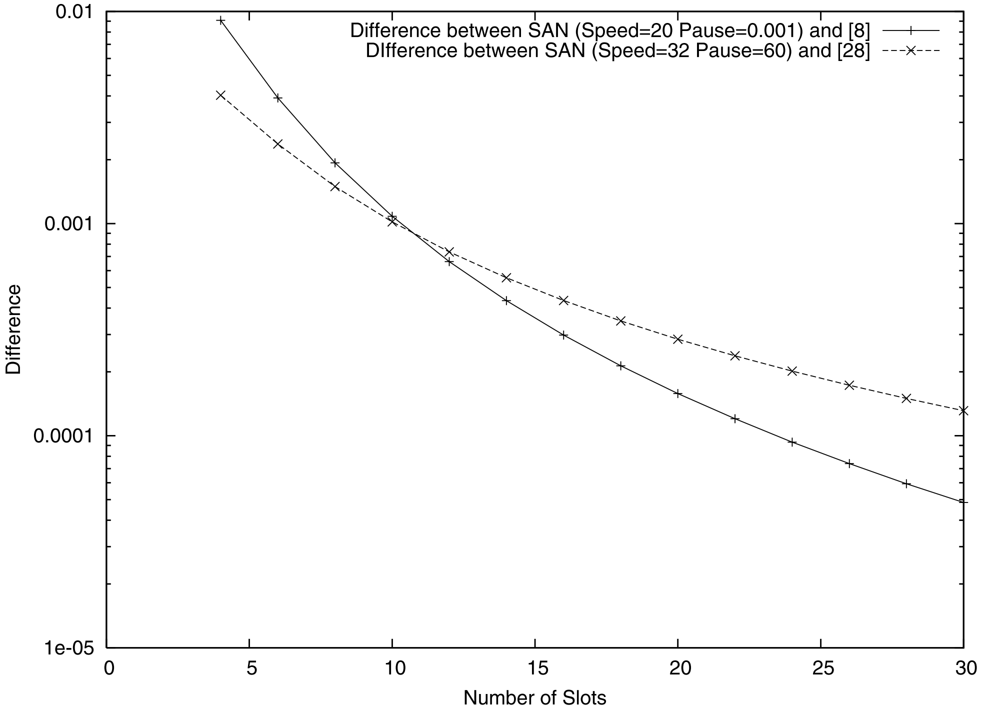


Fig. 4 Difference between continuous space theoretical results and SAN model results

if $x \in [0, 1]$, and $f(x) = 0$ otherwise,

$$\text{where } p = \frac{Pause}{Pause + \frac{1}{3v}} \tag{2}$$

Additionally, (2) expresses the probability that a node remains stationary during the whole experiment (p_{stat}), that was not considered for our comparison (we assume $p_{stat} = 0$).

The first validation is a simple comparison with theoretical results expressed by (1) using $a = 20$, i.e., our model with twenty 50 m slots ($M = 1{,}000$ m and $N = 20$), a very small pause time ($Pause = 0.001$ sec.), and node speed of 20 m/s ($rtSpeed = 20/50$). With this same number of slots, we change the pause time to 60 sec. and the speed to 32 m/s and compared our results with those given by (2). Note that as the size of area used by (2) is 1, the speed (v) used in this equation must be brought in proportion, then we divided the speed in m/s by 1000, which is the size of our area (M).

Numerically speaking, the obtained results have shown the same slot probabilities with a difference around 10e−4. We also tested some small changes in the pause time (from 0.1 to 0.00001 sec.) without having significative changes of probability difference. On the contrary, the choice of the number of slots (N) keeping the same area (M) changes the results considerably. Figure 4 shows that assuming from 4 to 30 slots, the difference between probabilities computed by our discrete space model and probabilities found by [7] and [46] theoretical results considering continuous space decreases exponentially until near 10e−5.

Figure 5 presents the probability distribution among 20 slots of a 1,000 meters region analyzing the node mobility with a 32 meters per second speed and pause times of 0.001, 4, 64, and 256 seconds. We can observe that as the pause time increases the node spatial distribution spreads more equally in the region, i.e., the result tends to the uniform distribution. This is expected since when the pause time increases, the time spent on the path becomes less important than the pause time in the spatial node probability. Since the destination is chosen equiprobably, as the pause time increases, the spatial node probability tends to be uniformly distributed.

Another approach analyzed is the node distribution with different speed values. According to [7] and [46], the speed does not affect the distribution considering a zero pause time. Even though we could verify this behavior, we observe that for a more significant pause time ($pause = 60$ sec.) the probability distribution was quite different for different speeds (1 m/s, 4 m/s, 16 m/s, and 64 m/s). In fact, Fig. 6 shows a similar effect as observed in varying the pause time, i.e., the more relevant the pause time is in comparison to the node speed, the more close to an equiprobable distribution the result will be. Those results were also consistent with the conclusions presented in [46].

4.2 Random waypoint 2D SAN model

The second proposed SAN model represents the region modeled in a two-dimensional space, e.g., a surface where

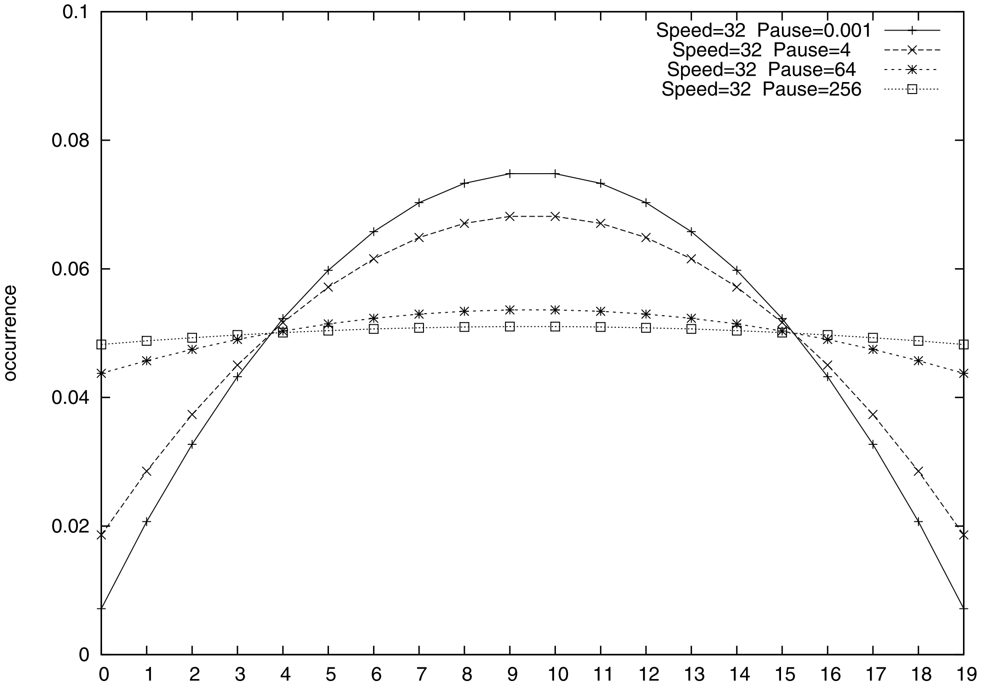


Fig. 5 1D random waypoint spatial node distribution—varying the pause

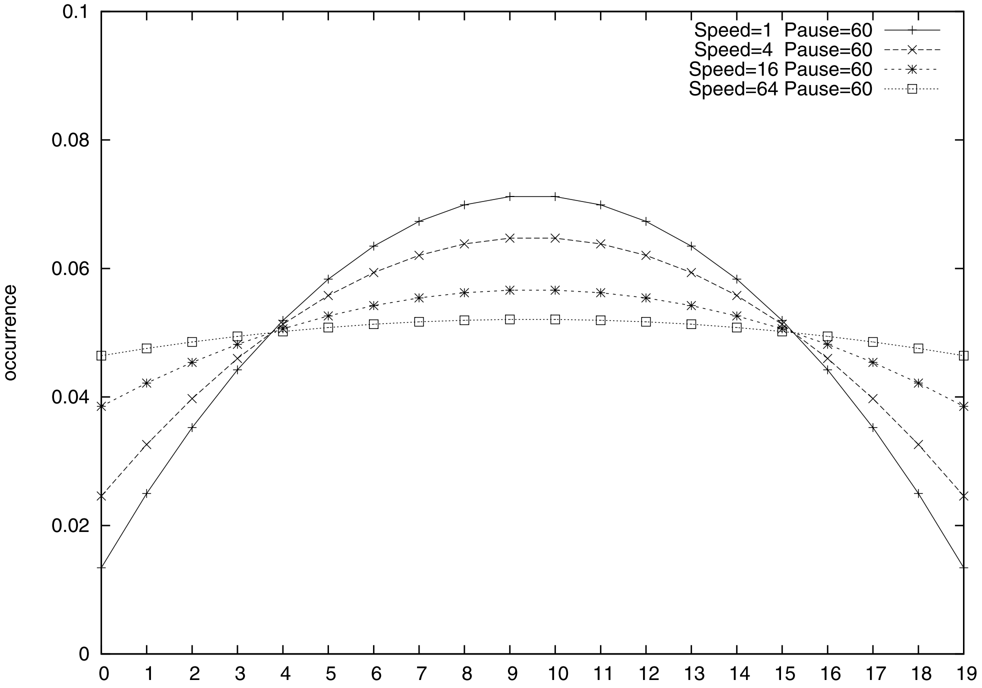
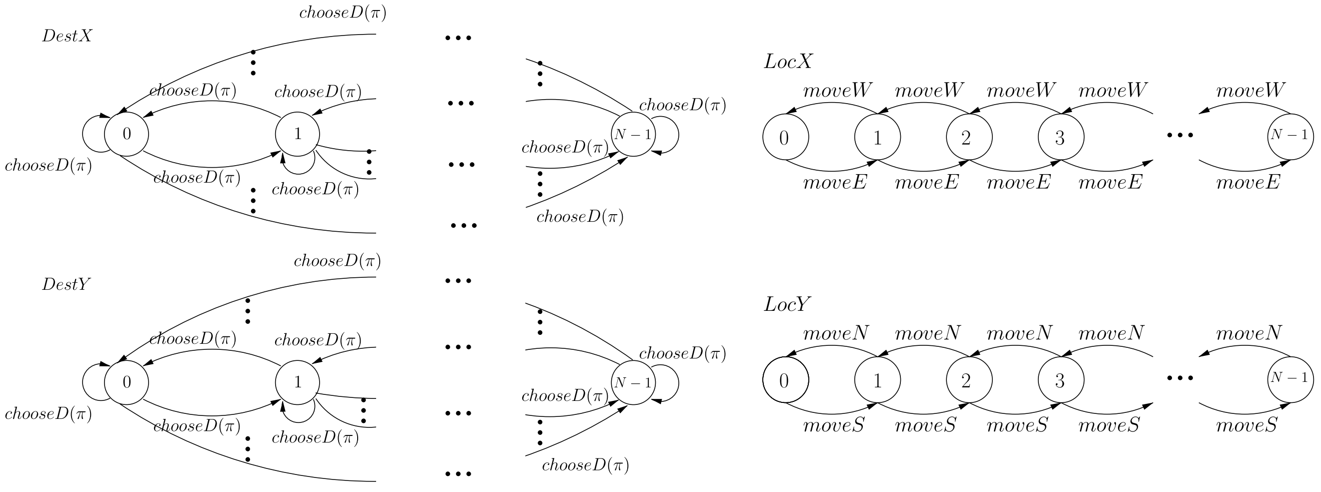


Fig. 6 1D random waypoint spatial node distribution—varying the speed

one node can move to north, south, east, and west. The model representing this 2D movement is presented in Fig. 7. This model is obtained through a natural extension from the 1D model of the previous section. The same structure for 1D is replicated, obtaining the four automata that compose the corresponding SAN to the 2D model. In this model, there are two automata to describe the node destination point (*DestX* and *DestY*) and two automata to describe the current node location (*LocX* and *LocY*). In both pairs of automata, a point is obtained by a x- and y-coordinate, i.e., the surface is split in $N \times N$ slots.

Even though similar, the behavior of the transitions and events in the 1D model has few differences in the 2D model. The first one is that the choice of the next destination (event *chooseD*) is now represented by a synchronizing event. The change of destination can change both x- and y-coordinates so this may represent the occurrence of transitions in both *DestX* and *DestY* automata. Despite of that distinct repre-

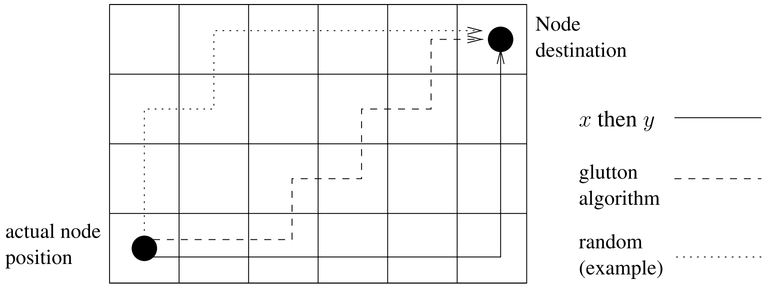


Type	Event	Rate
loc	$moveW$	f_W
loc	$moveE$	f_E
loc	$moveN$	f_N
loc	$moveS$	f_S
syn	$chooseD$	f_D

$$f_W = rtSpeed * ((stDestX) < (stLocX))$$
$$f_E = rtSpeed * ((stDestX) > (stLocX))$$
$$f_N = rtSpeed * ((stDestY) < (stLocY))$$
$$f_S = rtSpeed * ((stDestY) > (stLocY))$$
$$f_D = (1/Pause) * (((stDestX) == (stLocX))\&\&((stDestY) == (stLocY)))$$
$$\pi = 1/N$$

Fig. 7 Random waypoint 2D SAN model

Fig. 8 Example of routing strategies



sentation (*chooseD* was local for the 1D model), the other characteristics of event *chooseD* are preserved, namely its rate expressed by function f_D and its probability expressed by π. Note that, function f_D has an analogous way to express if the node is in a *pause* situation, since it has now to consider the same state in both x and y axes, i.e.,

$$\begin{aligned} f_D = {} & (1/Pause) \\ & * \big(\big((stDestX) == (stLocX)\big) \\ & \&\&\big((stDestY) == (stLocY)\big)\big) \end{aligned}$$

Another difference between 1D and 2D models is the way to represent the node movement. In the 2D model, the movement is represented by four local events describing movements toward north, south, west, and east directions (respectively, *moveN*, *moveS*, *moveW*, and *moveE*). The rates for those events remain computed as in the 1D model, i.e., according to the node speed and the slot size. Nevertheless, the expression of this movement event rates needs a more elaborated definition to take routing strategies into account.

The major difference between the 1D and 2D models resides in this choice of routing strategy. It is necessary to define whether a node movement may happen in the x- or in the y-axis. Note that this need for routing strategy definition is only an important matter due to our model assumption of a node moving in a discrete space. It is clear that the probability distribution on slots may change according to the chosen routing strategy. For instance, it is intuitive to expect different probability distribution results to a *first x then y routing*, a *glutton algorithm routing*, or even a *random routing* (Fig. 8).

The simplest routing strategy to represent in our SAN model is the *random* decision. In that case, we let the choice of axis movement be stochastically decided between two possible events. The model of Fig. 7 represents this option. Events rates only express if the movement is necessary or not, e.g., event *moveW* is conditioned only by the state of *DestX* being smaller than the state of *LocX*.

The representation of a *first x then y routing* strategy is a little bit more elaborated, since it represents a restriction in the y-axis movements. It will correspond to modify the functional rates of $moveN$ and $moveS$ (y-axis) events to include the restriction to move only if a x movement is no longer needed. The functional rates for events $moveN$ and $moveS$ respectively should be rewritten as follows:

- $f_N = rtSpeed * (((stDestY) < (stLocY))\&\& ((stDestX) == (stLocX)))$
- $f_S = rtSpeed * (((stDestY) > (stLocY))\&\& ((stDestX) == (stLocX)))$

The representation of the *glutton algorithm* strategy is even more elaborated. It allows a node to move only if this movement is in the *farthest* direction to the destination. The four functional rates for events $moveW$, $moveE$, $moveN$, and $moveS$ should be rewritten as follows:

- $f_W = rtSpeed * (((stDestX) < (stLocX))\&\& (((stLocX) - (stDestX)) >= (max((stDestY) - (stLocY), (stLocY) - (stDestY)))))$
- $f_E = rtSpeed * (((stDestX) > (stLocX))\&\& (((stDestX) - (stLocX)) >= (max((stDestY) - (stLocY), (stLocY) - (stDestY)))))$
- $f_N = rtSpeed * (((stDestY) < (stLocY))\&\& (((stDestY) - (stLocY)) >= (max((stDestX) - (stLocX), (stLocX) - (stDestX)))))$
- $f_S = rtSpeed * (((stDestY) > (stLocY))\&\& (((stLocY) - (stDestY)) >= (max((stDestX) - (stLocX), (stLocX) - (stDestX)))))$

None of the discrete space routing strategies will precisely describe what happens in a continuous space movement. However, the best approximation of the continuous movement behavior is achieved by the *glutton algorithm* strategy, specially considering a rather large number of slots.

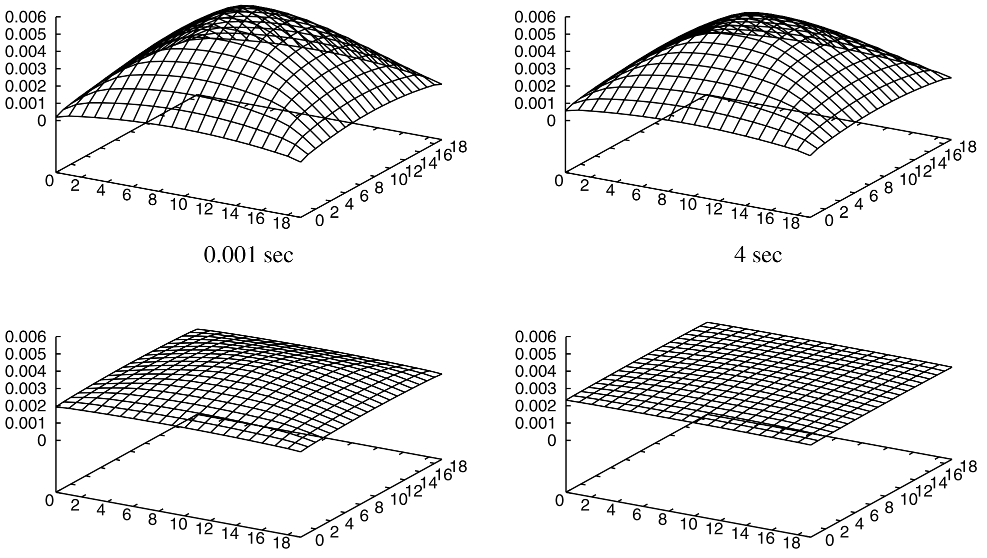
4.2.1 Validating the 2D model

Analogously to the 1D model, the validation of the 2D can be done by applying it to similar cases presented in [7] and [46]. For that matter, we also analyze a near zero pause time (0.001 sec.) model with 10 meters per sec. speed, a surface of 1000 × 1000 square meters split in 20 × 20 slots, and the *glutton routing* strategy. Once again the comparison of theoretical results presented in [7, 46] with our model's presented a difference around 10e−4. Also, using the same parameters with a different slot granularity (5 × 5 and 10 × 10 slots), we observed the difference decreasing consistently with the 1D comparison (respectively, 10e−2 and 10e−3).

4.2.2 New results for the 2D model—pause vs. speed

The variations of pause time and speed already made for the 1D case (Figs. 5 and 6) were reproduced with the 1000 × 1000 surface split in 20 × 20 slots model also using the *glutton algorithm routing* strategy. Figure 9 shows the probability distributions obtained for models with a fixed speed (32 m/s) and varying the pause time for 0.001, 4, 64 and 256 seconds. Analogously, Fig. 10 shows the results for fixing the pause time in 60 sec. and varying the speed for 1, 4, 16, and 64 m/s. Once again, these results show a very similar behavior as the one found for the 1D models, i.e., as the relevance of the pause time increases in comparison with the node speed, the probability distribution tends to be uniform.

Fig. 9 Random waypoint spatial node distribution in 20 × 20 area—varying the pause



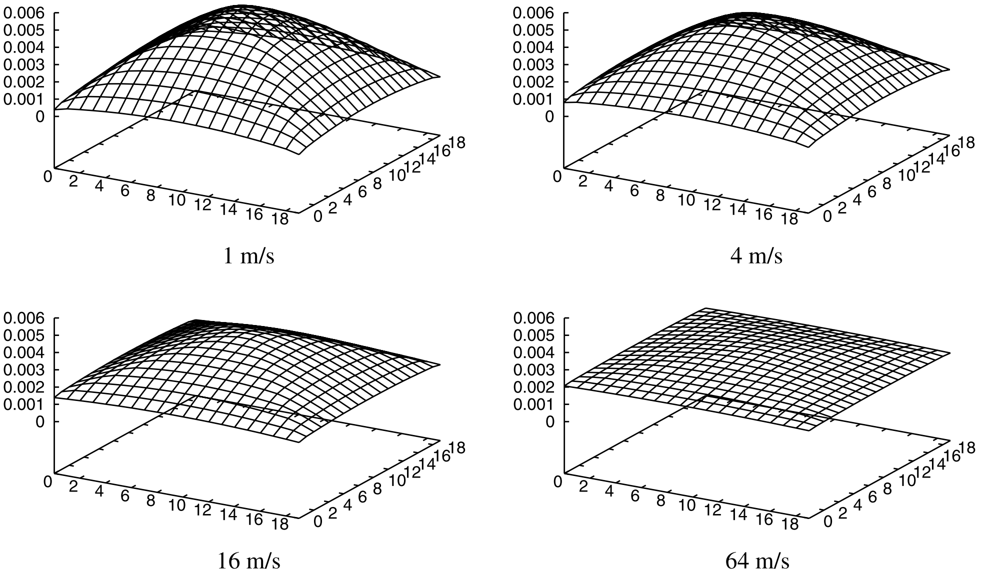
0.001 sec

4 sec

64 sec

256 sec

Fig. 10 Random waypoint spatial node distribution in 20 × 20 area—varying the speed



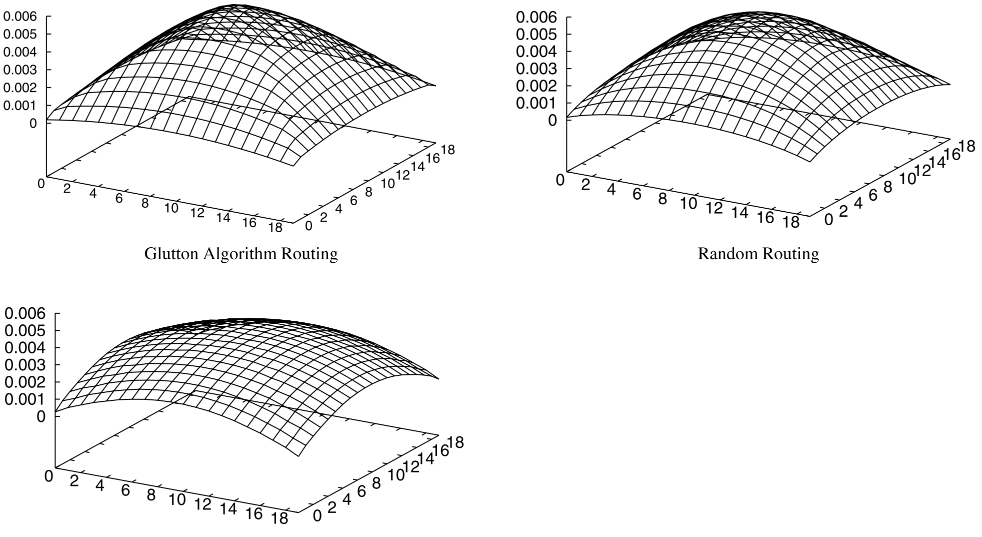
1 m/s

4 m/s

16 m/s

64 m/s

Fig. 11 Random waypoint spatial node distribution in 20 × 20 area—routing effect



Glutton Algorithm Routing

Random Routing

First X then Y Routing

4.2.3 New results for the 2D model—routing strategies

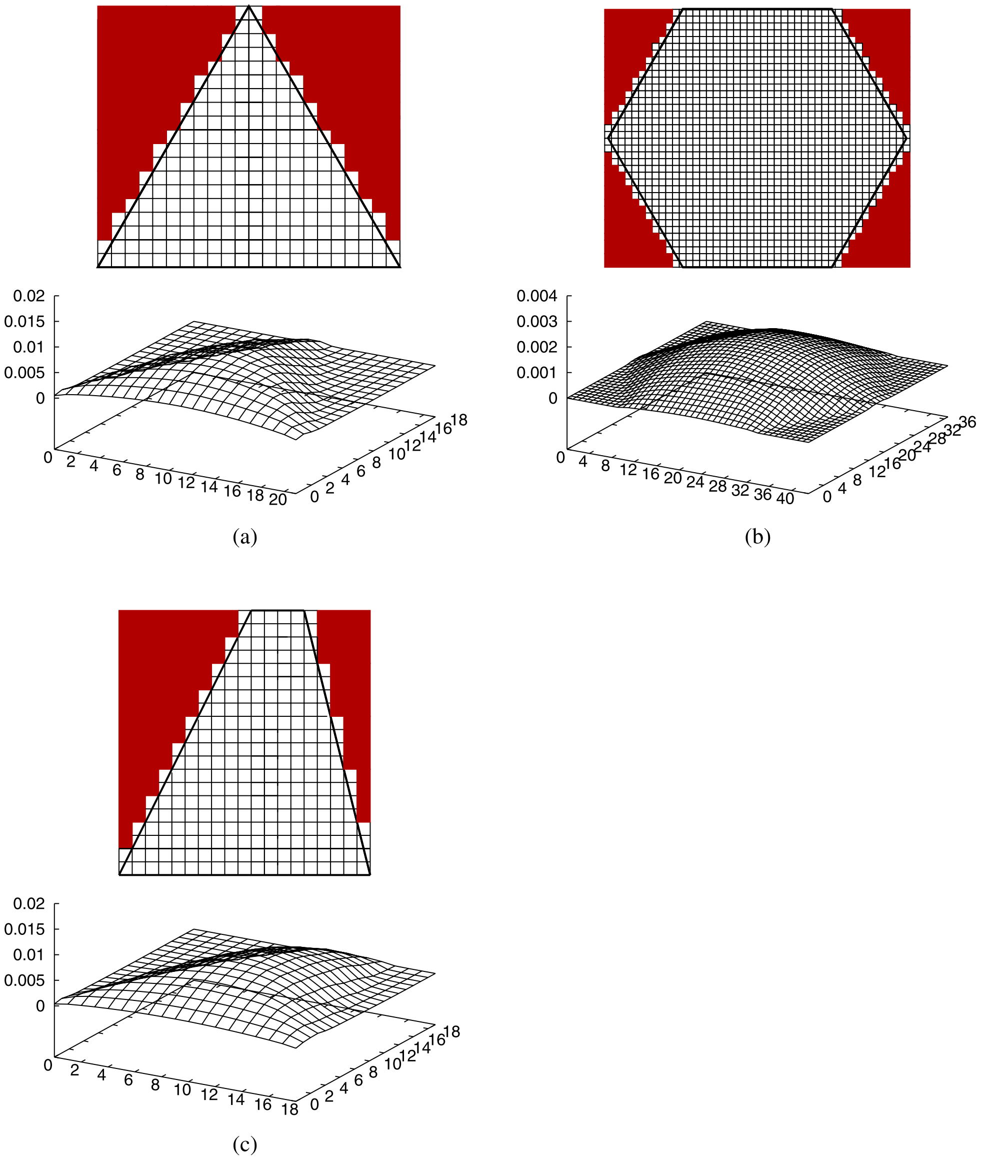
Figure 11 shows the probability distributions found for the three strategies presented in the previous section (*glutton, random,* and *x then y*) for a model with 1000 × 1000 square meters surface split in 20 × 20 slots, node speed of 5 m/s and pause time of 0.001 sec. In opposition to the other experiments when a deformation toward equally distributed results were achieved without changing the curves shape, those routing strategies variations do change the distribution shape. It is natural that the greatest effect was found for the *x then y* strategy, since this choice of routing tends to avoid the central node positions.

4.3 Random waypoint 2D modeling of non-square surfaces

An important feature of SAN is the modular construction that helps the reuse and the replacement of model parts. In this section, we present the results of modeling different convex areas. The proposed SAN model is suitable to describe also areas such as triangles, hexagons (as in [29]), and virtually any polygon. The results obtained to all irregular surface models tested have shown consistent behavior to those previously obtained for square surfaces (Sect. 4.2).

In these models, we consider a square envelope surface and we add obstacles on the borders. In Figs. 12a, 12b, and 12c, the unreachable spots are represented by filled po-

Fig. 12 Random Waypoint in a regular and irregular surface—spatial node distribution. (**a**) Triangular area. (**b**) Hexagonal area. (**c**) Trapezium area



(a)

(b)

(c)

sitions, while reachable spots are represent by blank positions. Note that due to the surface discretization, the reachable slots do not correspond exactly to the expected polygon, but merely to an approximation as close as possible.

The unreachable slots modeling is two-fold: functions in event rates prevent or allow nodes to move depending of x and y-axis in the automata *LocX* and *LocY*; and synchronizing events in automata *DestX* and *DestY* prevents the choice of a new destination to an unvalid position (an unreachable slot).

The routing strategy used for nonsquare models is a variation of the glutton algorithm, but a slight change was done to prevent node blocking, which may happen in very particular cases near an irregular border.

4.3.1 New results for the 2D model—convex polygon surfaces

The modeled areas are presented in the upper part of Fig. 12 and their corresponding spatial node distributions are presented in the lower part of the same figure.

To represent a regular triangle we assume 22 slots as base and hypotenuse, and 19 slots as height. The results of the spatial node distribution inside this triangular surface were obtained considering each slot with 50 meters, as in the square area, with node speed of 32 m/s and pause time of 0.001 sec.

Analogously to the experiments presented in Sect. 4.3, variations were performed for the triangular surface, fixing node speed in 32 m/s and varying the pause time for 0.001,

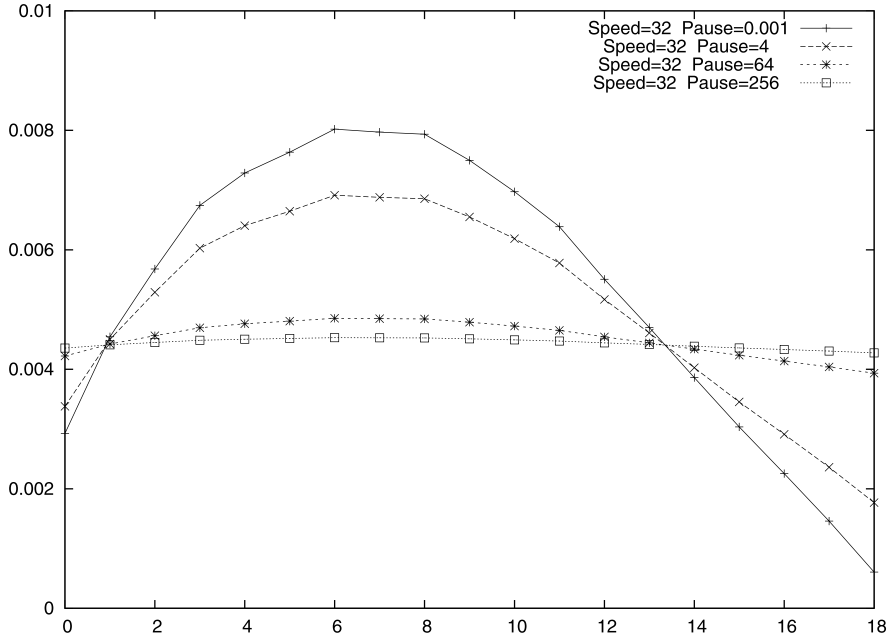


Fig. 13 Cross section in middle y-axis of triangle surface—varying pause

4, 64, and 256 seconds. Figure 13 shows the probability distributions in the cross section along the middle y-axis of the triangle. Similar experiments were performed for a pause time fixed in 60 seconds and speed varying in 1, 4, 16, and 64 m/s, and the results show the same behavior presented in Sect. 4.2.1, i.e., as concluded for square models, the proportion between pause and speed values seems to be the major drive for the spatial distribution.

Additionally, as validation of our modeling choices, we observe that the curve representing the node distribution for the middle y-axis section with speed in 32 m/s and pause time in 0.001 seconds presents a behavior very close to the results presented by Hyytia et al. in [29].

In the hexagonal area, we consider the dimensions of a regular triangle to each sixth part that composes the hexagon. Then the hexagonal area was described inside a rectangular envelope of 44 × 38 slots. The results presented in Fig. 12b were obtained with speed in 32 m/s and pause time in 0.001 seconds.

These same speed and pause values were considered in the results of the trapezium shaped surface (Fig. 12c), that lies inside a 19 × 20 slots envelope. This last example intends to stress that unlike the previous results of the literature [29], also irregular surfaces can be modeled as well.

It is also important to observe that the modeling of other surfaces became elegant and natural, needing mainly to specialize the definition of the movement and destination choice transitions, but keeping the main parts of the generic square 2D model. Thus, it is possible to model more realistic surfaces and to perform predictions about their spatial node distribution. In fact, even non-convex polygons could be modeled, but for these cases a more careful definition of the routing strategy functions is needed.

5 Random direction

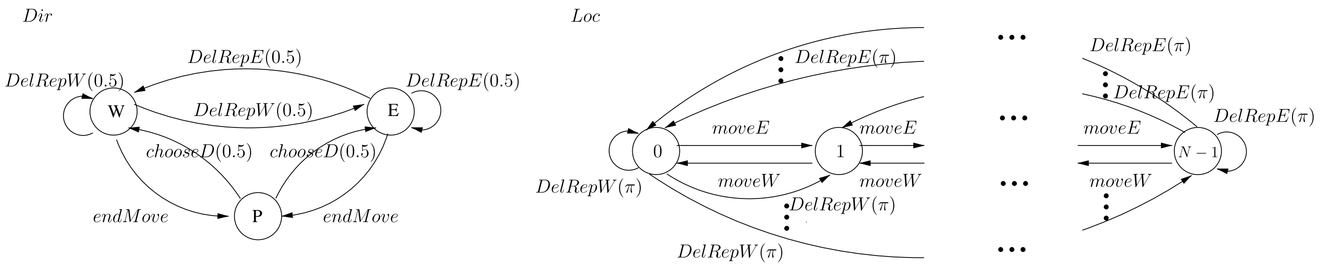
In this section, we assume the Random Direction mobility model as described by Guerin et al.in [25]. Our analysis of the Random Direction mobility model with *Bounce* and *Wrap Around*, using SAN, resulted in a completely equiprobable spatial node distribution, which is coherent with the literature [5]. Therefore, we concentrate on the Random Direction mobility model with the *Delete and Replace* border behavior, where non-equiprobable results were found.

Considering that, the behavior of a node is modeled as follows: the node randomly chooses a direction to move with a constant speed, and, after moving during an also randomly chosen duration time, the node makes a *Pause*, then a new choice of direction is done starting this process over and over.

Most of the modeling definitions of the Random Waypoint example were kept for both the one- and two-dimensional versions of the Random Direction models. Namely, we still divide the surface of interest of size M into N slots, we split the information of node location and its movement in different automata, we model the movement speed and pause duration with rates *rtSpeed* and *Pause* from the same input parameters information.

5.1 Random direction 1D SAN model

The model presented in Fig. 14 is composed of 2 automata, called *Dir* and *Loc*. Automaton *Dir* represents the randomly chosen direction. There are three basic situations for a node in such SAN model:



Type	Event	Rate
loc	$moveE$	f_E
loc	$moveW$	f_W
loc	$chooseD$	f_D
loc	$endMove$	f_M
syn	$DelRepW$	f_{DR}
syn	$DelRepE$	f_{DR}

$f_E = (stDir == E) * rtSpeed$
$f_W = (stDir == W) * rtSpeed$
$f_D = 1/Pause$
$f_M = (Speed/MoveSize)||(1/MoveTime)$
$f_{DR} = rtSpeed$
$\pi = 1/N$

Fig. 14 Random direction 1D SAN model

(i) The node is stopped, represented by state P in automaton *Dir*. In this case, the node does not move, i.e., automaton *Loc* does not change its local state, and a choice of direction is the next action. This choice of direction is represented by event *chooseD*, changing the local state of *Dir* automaton to states W or E with the same probability.

(ii) The node is moving and it is not yet on a border slot, i.e., either automaton *Dir* is in state W and automaton *Loc* is in the range $1..N-1$, or automaton *Dir* is in state E and automaton *Loc* is in the range $0..N-2$. In this situation, if automaton *Dir* is in state E, then only the event *moveE* is enabled, representing the movement to East and, of course, the movement toward West is analogous. While moving, the node may stop at any time and go to the *pause* situation. Such change of situation is represented by the event *endMove* from states E or W to state P in automaton *Dir*.

(iii) The node is moving and it is on a border slot, i.e., either automaton *Dir* is in state W and automaton *Loc* is in state 0, or automaton *Dir* is in state E and automaton *Loc* is in state $N-1$. In this case the node may either stop moving, represented by the possibility of *endMove* as in the previous situation, or the node may actually cross the border, and then be deleted and replaced by a new node that can be created in any location. This second possibility is represented by events *DelRepE* and *DelRepW* according to the direction the node was moving toward (respectively, East and West).

The deleted-and-replaced node continues to move (no pause) and a new choice of direction is made together with the replace, meaning that after a replace the node may move to a different direction. In automaton *Dir*, the choice of the same or a different direction is represented by the two possible (and equiprobable) occurrences of events *DelRepW* from state W and *DelRepE* from state E. That being said, we also analyze a variant of this behavior in which the node keeps its direction after a delete-and-replace, but the results obtained for both possibilities were exactly equal. In automaton *Loc*, the occurrence of a delete-and-replace is also represented by either event *DelRepW* or *DelRepE* and it always change from state 0 or $N-1$ (respectively) to any state.

As said before, the rate for events in the Random Direction model were computed almost always like they were for the Random Waypoint model. This is the case of the *chooseD* rate that still is the inverse of the pause time, but, unlike the Random Waypoint model, here it does not depend on reach a destination point, i.e., $f_D = \frac{1}{Pause}$.

Also, the east and west movement rates are calculated according to the numerical value of *rtSpeed* tempered by the necessity of the specific direction movement:

$$f_W = (stDir == W) * rtSpeed$$
$$f_E = (stDir == E) * rtSpeed$$

The event *endMove* rate is computed as the inverse of the sojourn time of automaton *Dir* in alternatively state E or state W, i.e., the expected duration of a movement. This duration could be directly expressed by an input parameter *MoveTime* or alternatively by the expected distance of movement (*MoveSize*) divided by the node speed, i.e.,

$$f_M = \frac{1}{MoveTime} = \frac{Speed}{MoveSize}$$

The rates for events *DelRepW* and *DelRepE* are equal between themselves, and its numerical value is equal to the rate of the movement (*rtSpeed*). But unlike the movement events (*moveW* and *moveE*), there is no need of express a condition, since the *DelRep* have their occurrence already restricted by the synchronization between automata, i.e., events *DelRep* will only occur when the automata *Dir* and *Loc* are in one of two precise states (situation (iii) described previously in this section):

$f_{DR} = rtSpeed$

5.1.1 Validating the 1D model

To validate the proposed model we consider a 1000 meters region ($M = 1.000$) divided in 20 slots of 50 meters each, and vary the *pause* time, the speed, and the size of movement.

In the first experiment, we fix the size of movement in 50 meters and vary the speed for 1, 4, 16, and 64 m/s and pause times for 0.001, 16, 64, and 256 seconds. Figure 15 presents the spatial node distribution for the several combinations of those parameters.

In Fig. 15, we also repeat the same combinations of values of speed and pause time with size of movement of 200, 800, and 1600 meters. The first important conclusion obtained is that for all combinations of speed and pause time, fixing the size of movement, we observe precisely the same numerical results.

In fact, the variation between the amount of time that a node stays in a slot due to pause and due to movement shows complementary figures, regardless of how much pause time or node speed is assumed. Figure 16 demonstrates such behavior by drawing the total node location distribution for all node speeds and pause times combinations (curve T) considering a Size of Movement of 200 meters. The amount spent due to pause and movement for *Pause* = 4 and *Speed* = 4 (curves P and M) and for *Pause* = 4 and *Speed* = 32 (curves P′ and M′) are also plotted and one can notice that the results in curve T are equivalent to the sum of curves P and M, as well as the sum of curves P′ and M′.

Therefore, we conclude that *Pause* and *Speed* values do not influence the spatial node distribution results for this mobility model, using delete-and-replace as border behavior. In fact, the spatial distribution only depends on the size of movement. As the size of movement increases, the spatial node distribution approaches the uniform one.

Using this result we simplify the model presented in Fig. 14 replacing the *Dir* automaton (with three states) by a simpler automaton with only two states depicted in Fig. 17, where state *P* is absent. As said (Sect. 3), by reducing the number of automata, or the number of states inside an automaton, the model becomes easier to handle.

This new version of automaton *Dir* has three events called *chooseD*, *DelRepW*, and *DelRepE*. The events *DelRepW* and *DelRepE* have exactly the same function as before, i.e., they are used to model the border behavior.

The event *chooseD* has also the same semantics as before. However, in this case, as there is no *pause*, the new direction choice will be performed as soon as either the size of movement or the moving time finishes. The numerical solution for the model without pause, as expected, is consistent with the equivalent continuous space models with pause described in the literature. Numerically speaking, the difference found between our model with or without pause is quite small (around 10e−6 for a 20 slots model).

It is important to notice that the composition of different automata used in this 1D model, exemplifies one of the greatest advantages of using a structured Markovian formalism such as SAN. Namely, the change, replacement, or even

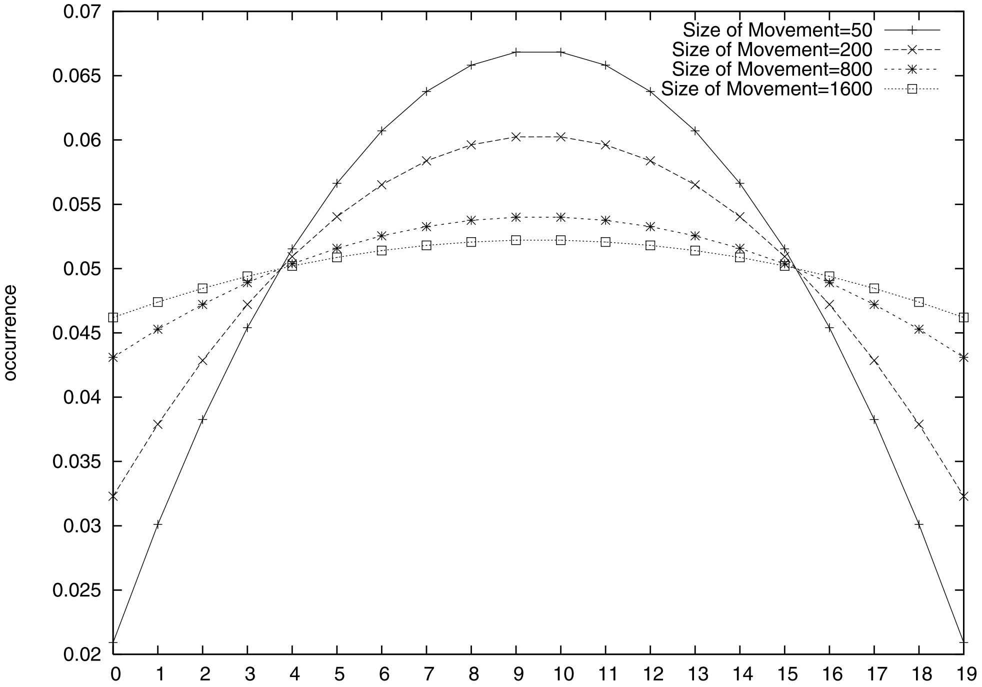


Fig. 15 Varying size of movement for all combinations of speed (1, 4, 16, 64 meters per second), pause time (0.001, 16, 64, 256 seconds) and size of movement (50, 200, 800, 1600)

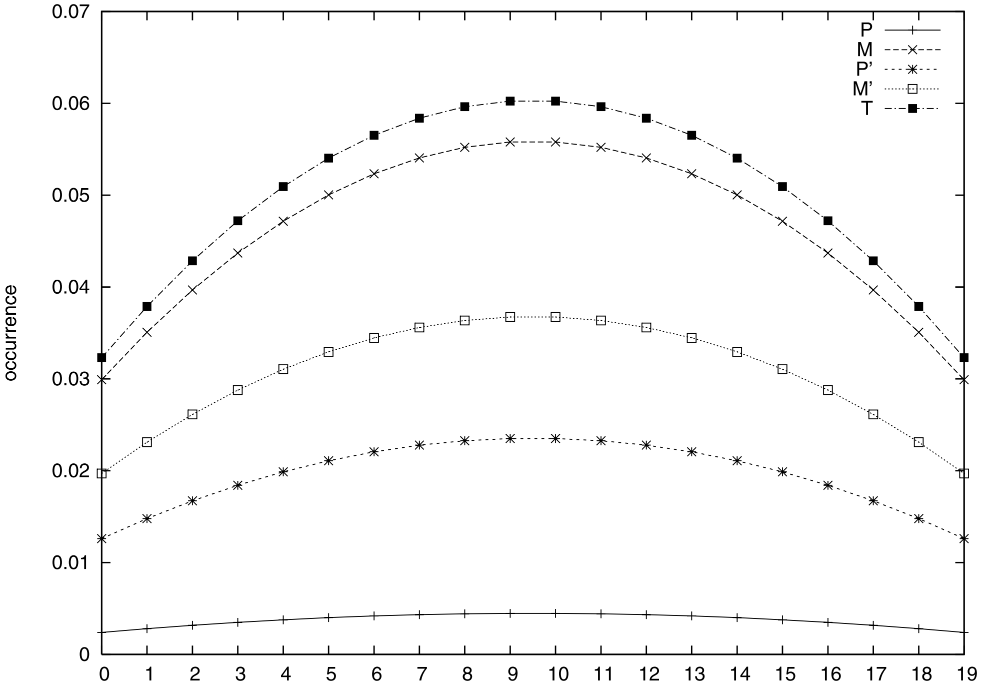


Fig. 16 Pause and movement components in spatial distribution probability

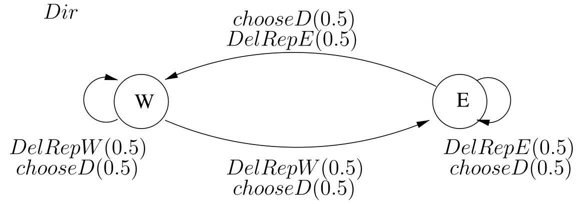


Fig. 17 *Dir* automaton for 1D model without *Pause*

Type	Event	Rate
loc	$chooseD$	f_D
syn	$DelRepW, DelRepE$	f_{DR}

$f_D = (Speed/MoveSize)||(1/MoveTime)$

$f_{DR} = rtSpeed/NumSlots$

removal of an automaton usually do not compromise and almost always do not even affect the definition of the other automata.

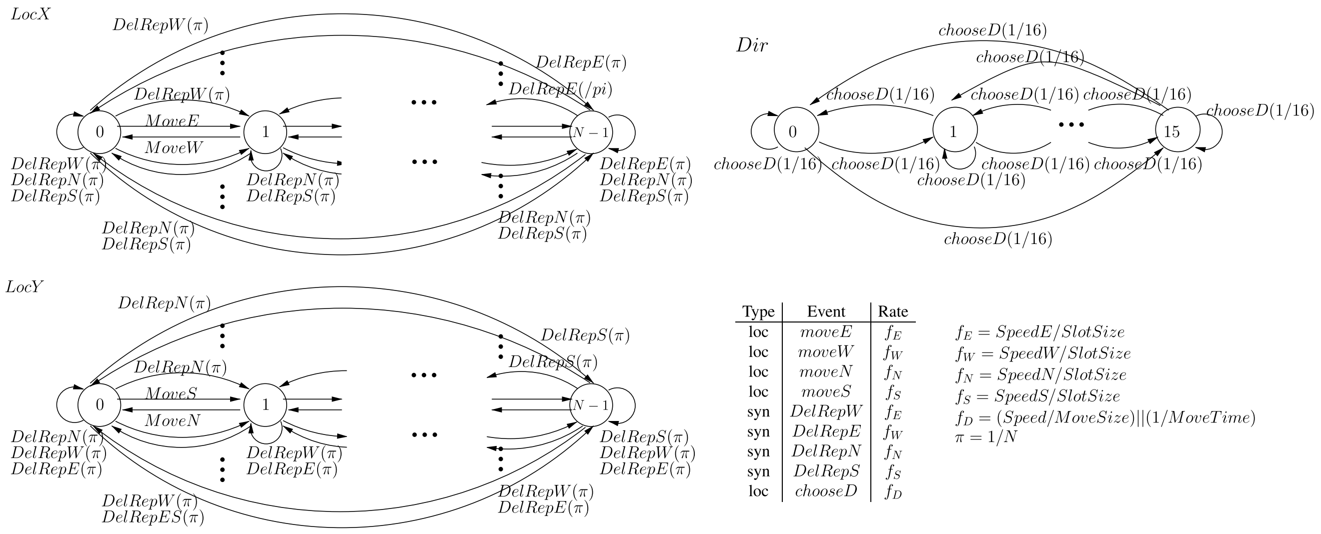
5.2 Random direction 2D SAN model

The second proposed SAN model for Random Direction represents a two-dimensional space (Fig. 18) and as in the Random Waypoint models, it is a clear extension of the 1D model presented in the previous section. That is the case of the location of the node that was represented by automaton *Loc* in the 1D model, and now is represented by two automata (*LocX* and *LocY*). The surface is divided in $N \times N$ slots and the current position of a node is a coordinate given by the local states of automata *LocX* and *LocY*.

After building and analyzing several 2D models, with different pause times and without pause, we observed that the pause time also does not influence the spatial node distribution in 2D models. This generalizes the results of the 1D model with respect to pause. In consequence, we replace the automaton *Dir* according to Fig. 17. Therefore, in the following, we discuss the 2D model without any consideration of pause. Analogously to the 1D case, this modification is modular and does not affect automata corresponding to the node location.

Although several ways can be employed to describe the node direction in a 2D model, the proposed model represents the possible moving directions with one automata (*Dir*), as well as in the 1D model. However, for the 2D case, *Dir* represents the possible angles that the node can move. The choice of the angle indicates implicitly the direction chosen (North, South, East, and West). Figure 19 represents the possible angles considering automaton *Dir* with sixteen states, i.e., sixteen discrete directions will be considered. Obviously, automaton *Dir* could have any discrete number of directions, and such number will define the number of states in automaton *Dir*. Figure 18 depicts this choice of sixteen states.

The choice of the next angle is represented by *chooseD* event. The rate assigned to this event is the same used in



Type	Event	Rate
loc	*moveE*	f_E
loc	*moveW*	f_W
loc	*moveN*	f_N
loc	*moveS*	f_S
syn	*DelRepW*	f_E
syn	*DelRepE*	f_W
syn	*DelRepN*	f_N
syn	*DelRepS*	f_S
loc	*chooseD*	f_D

$f_E = SpeedE/SlotSize$
$f_W = SpeedW/SlotSize$
$f_N = SpeedN/SlotSize$
$f_S = SpeedS/SlotSize$
$f_D = (Speed/MoveSize)||(1/MoveTime)$
$\pi = 1/N$

Fig. 18 Random direction 2D SAN model

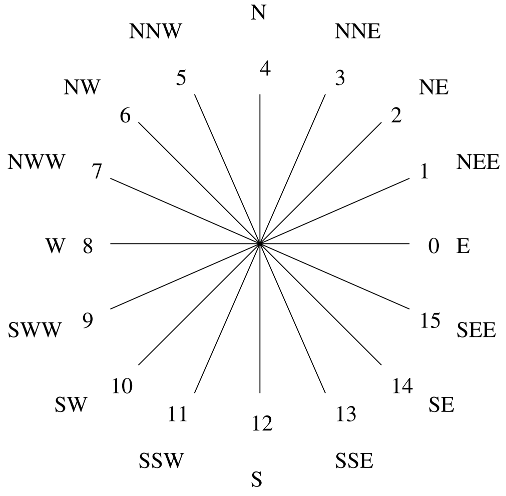


Fig. 19 Axis divided by sixteen directions

endMove event in the 1D model and it represents the time a node spends moving. Alternatively, and without any loss of generality, it can represent the distance described by a move step.

The actual move in a 2D area is decomposed in two components representing the move in X- and Y-axis. The rates of events *MoveE* (f_E) and *MoveW* (f_W) in automaton *LocX* and the rates of events *MoveN* (f_N) and *MoveS* (f_S) in automaton *LocY* are adjusted to represent these component speeds, and are represented below:

- $f_E = SpeedE/SlotSize$
- $f_W = SpeedW/SlotSize$
- $f_N = SpeedN/SlotSize$
- $f_S = SpeedS/SlotSize$

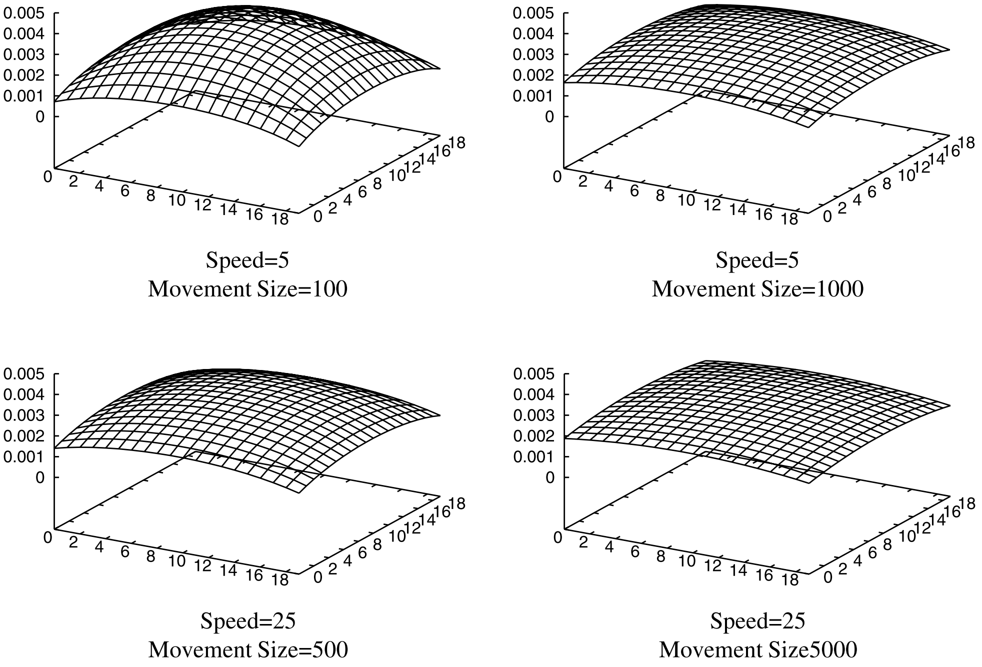
These functions depend on the node's average speed in meters per time unit and the size of the slot in meters (m/N). For instance, the east movement speed is given by the reason between the direction of the node movement (*MoveSizeE*) and the average time of it (*MoveTime*):

$$SpeedE = \frac{MoveSizeE}{MoveTime}$$

The movement direction (*MoveSizeE*) is decomposed according to the direction (in degrees). For instance, if a node is moving with speed Sp, to the northeast (NE), with 45 degrees of inclination (counterclockwise), then events *MoveE* and *MoveN* are both enabled and their rates are respectively calculated according to $Sp \times \text{cosin}(\alpha)$. Considering all possible directions that causes the east movement, the rate associated to *MoveSizeE* is:

$$\begin{aligned}
&MoveSizeE\\
&\quad = (((stDegree == 0) * (MoveSize))\\
&\qquad + (((stDegree == 1)||(stDegree == 15))\\
&\qquad * (MoveSize * 0.92387953251))\\
&\qquad + (((stDegree == 2)||(stDegree == 14))\\
&\qquad *(MoveSize * 0.70710678118))\\
&\qquad + (((stDegree == 3)||(stDegree == 13))\\
&\qquad *(MoveSize * 0.38268343236)))
\end{aligned}$$

The east movement consider the states 0 to 3 and 13 to 15 in automaton *Dir*, i.e., from SSE to NNE counterclockwise. This rate is decomposed according to the angle of movement, and has a weight for each degree, which is calculated with *cosin* formula. In this example, the weight increases as



Speed=5
Movement Size=100

Speed=5
Movement Size=1000

Speed=25
Movement Size=500

Speed=25
Movement Size5000

Fig. 20 Random direction spatial node distribution in 20 × 20 area and 16 directions—varying the speed and movement size

the angle approaches 0 degrees. Movement for other directions are similarly implemented.

When a node moves and reaches the border in one of the axis, the *Delete and Replace* behavior takes place. In both automata *LocX* and *LocY*, this behavior is modeled by events *DelRepN*, *DelRepS*, *DelRepE*, and *DelRepW*.

The border behavior defines that a new states in *LocX* and *LocY* have to be equiprobably chosen when one of the events occur. Taking automaton *LocX* for explanation, in any of its states it is possible to switch to any of the other states due to *DelRepN* and *DelRepS* since, regardless the horizontal position the node may be, these vertical reach border events may occur. In state 0, *DelRepW* can occur and, in state $N - 1$, *DelRepE* can take place depending if the node is moving to west or east respectively. This explains all possible events for each state. The explanation for automaton *LocY* is analogous.

5.2.1 Validating the 2D model

The results for the 2D model are consistent with those obtained for the 1D model and those presented in the literature [5]. They indicate that a more pronounced distribution is in the center area as the movement size or duration are shorter. Analogously, the node distribution tends to be uniform, as the movement duration or size grow. Figure 20 presents the spatial node distribution for a model of surface of 1000 × 1000 meters discretized in 20 × 20 slots with the same parameters of speed and size of movement presented in [5]. The values of speed considered were 5 and 25 m/s, and movement size considered was 100, 500, 1,000, and 5,000 meters.

However, the comparison with the results in the literature [5] demonstrate that our model gives a little flatter spatial node distribution, i.e., our predictions seems to have a less pronounced probability to find a node in the central area. We believe that this happen due to the number of directions considered (sixteen).

5.2.2 New results for the 2D model—varying the number of directions

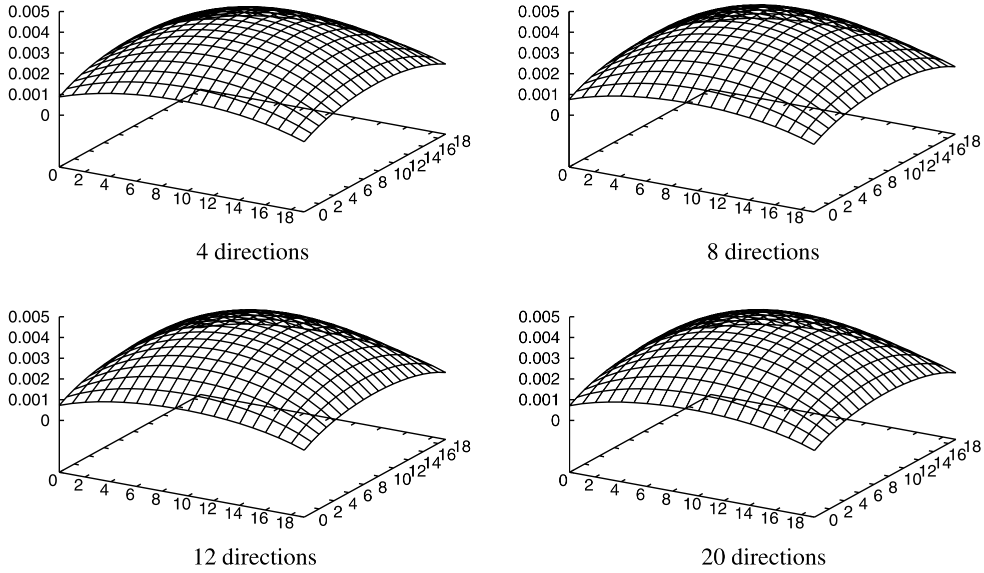
It is important to remind that our model assumes a discrete number of directions. Figure 21 depicts the variations in the node distribution when smaller or greater (4, 8, 12, and 20) number of directions are considered. The first plot of Fig. 20 complements Fig. 21 with information for 16 directions.

A closer look in these results is presented in Figs. 22 and 23. They present middle sections on the surfaces corresponding to the results with 4, 8, 12, 16, and 20 directions, where Fig. 22 shows these sections in the 5 central slots and Fig. 23 shows these sections for slots in the periphery of the 20 × 20 area. As can be observed, as the number of directions increases the impact on the results becomes less significant.

6 Conclusion

In this paper, we have formally describe the Random Waypoint and Random Direction mobility models using Stochastic Automata Networks (SAN). The results were validated

Fig. 21 Random direction spatial node distribution in 20 × 20 area with Speed = 5 and Movement Size = 100—varying the number of directions



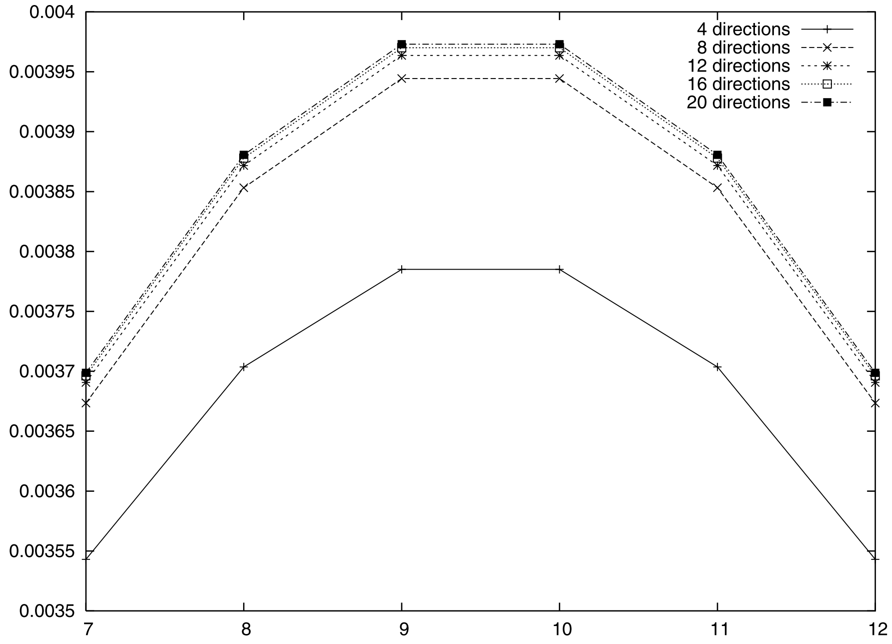
4 directions

8 directions

12 directions

20 directions

Fig. 22 Middle sections on the surfaces corresponding to the results with 4, 8, 12, 16, and 20 directions—5 central slots



by showing their numerical compatibility with existing analytical or simulation studies to the closest cases, i.e., the cases where a continuous surface was considered. Besides showing the suitability of SAN to the modeling of these mobility models, we have contributed to the better understanding of detailed aspects in the Random Waypoint such as pause time, node speed, and routing strategies in the 2D node distribution, especially considering a discrete mobility space.

We would like to stress the fact that all results presented in this paper were obtained by numerical solution of the SAN models, and not by any simulation technique. This fact enhances the contribution of this paper since the SAN modeling delivers more reliable results from a statistical point of view.

In the Random Direction, we have contributed to show that spatial node distribution is influenced by movement duration and that the speed and pause time do not influence the distribution. We also noticed that the (discrete) number of directions a node can move is relevant to the spatial distribution.

In our analysis, we obtained the spatial node distribution of a mobile node in a given area of M meters (1D) or M^2 square meters (2D) formed by a discrete number of slots. We observe that the granularity of this discretization has an impact on the spatial distribution results.

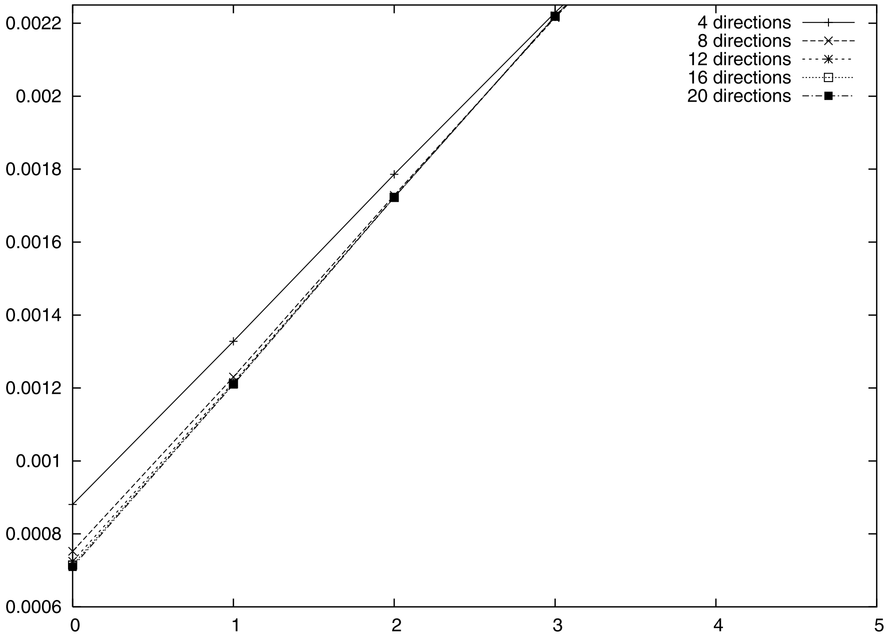
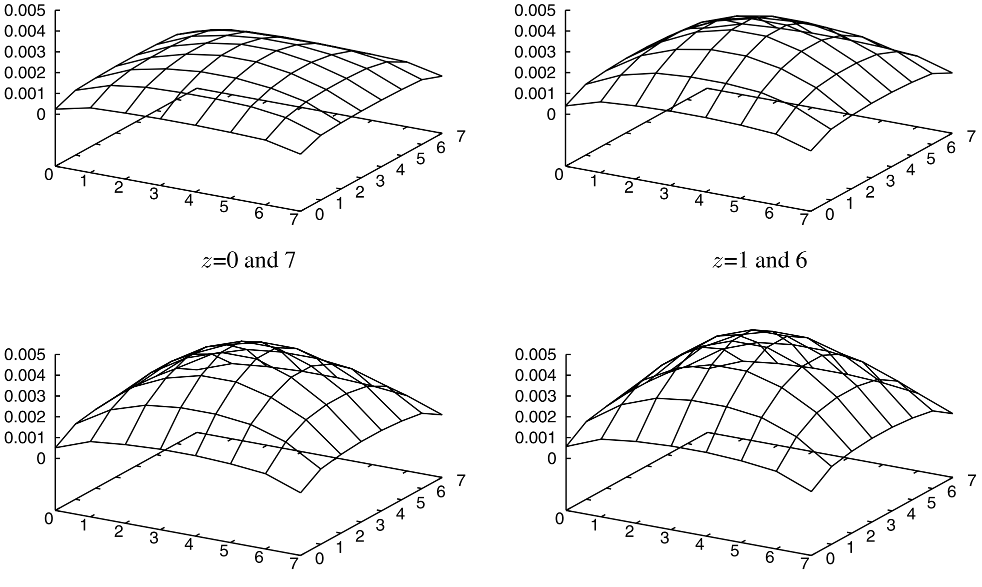


Fig. 23 Middle sections on the surfaces corresponding to the results with 4, 8, 12, 16, and 20 directions—peripheryc slots



z=0 and 7

z=1 and 6

z=2 and 5

z=3 and 4

Fig. 24 Random waypoint spatial node distribution in 3D

It is important to observe that we could modularly extend the SAN models to consider much more detailed additional aspects such as varying speed according to acceleration, slower/faster areas, or different pause time per position could be easily modeled. Actually, any other nonuniform behavior could be considered. Moreover, since we use automata networks, we can step-wise include more dimensions adding more automata and also adding some complexity to functions. Experimental three-dimensional space with Random Waypoint mobility model was described including two more automata to the 2D model. This last experiment considers a $8 \times 8 \times 8$ slots describing a space of 1 km^3, near zero pause time (0.001 sec.) and 10 meters per sec. speed. Figure 24 represents the spatial node distribution for the discrete slices in z axis. As expected, we found a higher probability of the node to be in the central slots of the solid region, i.e., in the center of the slices surfaces and in the central slices ($z = 3$ and 4).

This paper also contributes illustrating models for more complex areas than squares. We show that, with a Markovian formalism as SAN, we can model any regular or irregular polygon surfaces, but even solid structures can be envisaged. The modeling limits will be established by the state space explosion problem which is common to any Markovian formalism. In the future, we intend to model more complex realities, such as buildings with obstacles. In fact, obstacles can be suitably modeled in SAN through functional rates assigned to the transitions and reachability functions that limit reachable spots.

Natural future works, besides the ones already mentioned, are the use of the spatial node distributions achieved to obtain performance and dependability levels of such networks. Moreover, some numerical studies could be carried out to achieve more efficient spatial distribution predictions. For example, we could study stochastic properties of nearly independent automata (automata without synchronizing events) in order to derive product-form solutions, or at least solutions numerically more efficient. Such improved solutions could significantly reduce the computational cost to solve large complex models like the hexagon (Fig. 12c). This model, the largest solved within this paper, represents a 2.8 million states model, and it takes about a day to be solved in a small workstation, i.e., Xeon 2.2 GHz with 512 Mbytes cache and 2 Gbytes of RAM memory. Smaller models, like the Random Waypoint with 20 × 20 slots (a model with 160 thousand states) are usually solved in less than 30 minutes in the same machine. Recent improvements in dealing with structured models [16] could significantly reduce this solution times, but even with current software [10] the SAN models already offer valid insights with an affordable computational cost for quite large examples.

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