

## ANALYTIC METHOD FOR SIZING STAND-ALONE PV SYSTEMS IN BRAZIL

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**ABSTRACT:** Good performance of a stand-alone system is related to correct sizing. The purpose of this paper is to present an analytic method for sizing stand-alone systems in Brazil. The task of sizing a PV system consists of finding the PV-array area and the battery capacity that fit in well with the energy demand and the behaviour of solar radiation for a specific loss of load probability (LLP). To achieve this goal, a computer program was developed to simulate a stand-alone system with constant demand during the year. The system was simulated during ten years in order to find the slope of the modules that leads to the lowest PV-array cost for  $LLP=10^{-1}$  and  $LLP=10^{-2}$ . The procedure was then applied to obtain the isoreliability curves, i.e. the PV-array capacity as a function of the storage capacity, for both LLPs. This approach was carried out for 144 Brazilian cities. Equations were fitted to the values of slope of the modules found for each place as a function of the latitude and clearness indexes. The isoreliability curves can be obtained if two parameters, denominated  $a$  and  $b$ , are estimated. Two methods are presented. The method A consists of finding the parameters  $a$  and  $b$  as a function of the latitude and in the method B these parameters are obtained as a function of the latitude and clearness indexes. In this way, a stand-alone system can be more easily sized in Brazil, starting from a set of equations. The proposed analytic methods were applied to size a stand-alone system in Torres (latitude = - 29.3°). For  $LLP=10^{-2}$ , the difference between the simulated and estimated values is lower than 10% and 6% when method A and method B are used, respectively.

**Keywords:** Stand-Alone PV System; Sizing.

### 1 INTRODUCTION

Good performance of a stand-alone PV system is related to the correct sizing. However, for many places irradiation data does not exist to design the system and it is usually sized intuitively. Consequently, many problems may appear for users. For instance, if the system is oversized it becomes expensive and if it is undersized, the produced energy is not as expected.

The methods based on the simulation of the system for a large period are a good approach for sizing the system. However, in many times this kind of computer program is not available and, on the other hand, it is not easy to use. Due to these facts, some analytic methods were developed [1]-[4]. The purpose of this paper is to present an analytic method for sizing stand-alone systems in Brazil. The task of sizing a PV system consists of finding the PV-array area and the battery capacity that fit in well with the energy demand and the behaviour of solar radiation for a specific loss of load probability (LLP).

### 2 SIMULATION OF A STAND-ALONE PV SYSTEM

#### 2.1 Estimation of Daily Irradiation

In this simulation, the input data includes: slope of the module  $\beta$ , loss of load probability, storage capacity, PV-array efficiency and monthly average daily irradiation on a horizontal surface.

A sequence of daily clearness indexes is generated by the Aguiar's Method [5] and by using the monthly average daily irradiation on a horizontal surface. Following this, the daily irradiation is split into its beam and diffuse components on a horizontal surface [6]. Starting from the daily total and diffuse irradiation, the hourly irradiation data on a horizontal surface are estimated, using the method proposed by Collares-

Pereira and the ratio of hourly diffuse to daily diffuse irradiation is obtained from Liu-Jordan model [6], [7]. The hourly diffuse irradiation on a tilted surface is calculated using the Perez's model [8]. Hourly solar irradiation data is used because the methods to estimate the hourly diffuse irradiation on PV-array is more accurate. Then, the daily irradiation is obtained starting from the hourly values.

#### 2.2 Performance of Stand-Alone PV Systems

The electric energy provided by the PV system relies on the natural behaviour of local solar radiation. Consequently, the system is related to a loss of load probability (LLP) [1]-[4], [9],[10] defined as the ratio of energy deficit to energy demand.

The PV-array capacity  $C_A$  [1], [2] and the storage capacity  $C_S$  [2] are defined related to average daily energy demand  $L$ :

$$C_A = \frac{\eta A G_{dm}}{L} \quad (1)$$

and

$$C_S = \frac{C}{L} \quad (2)$$

where,

$A$  = PV-array area;

$\eta$  = PV-array efficiency;

$G_{dm}$  = annual average daily irradiation on a horizontal surface;

$C$  = maximum energy that can be taken out of the batteries.

The storage capacity represents how many days the battery can supply the demand, without any energy from the PV-array. For instance, if  $C_S = 5$ , this means that the batteries have stored energy for five cloudy days.

In order to calculate LLP, the daily state of charge (SOC) of the batteries is simulated for each day, during ten years. The state of charge is the ratio of available to nominal capacity of batteries. At the end of day  $i$ , SOC is

obtained by doing a balance of energy consumed by users and supplied by PV-array, as follows:

$$\text{SOC}(i) = \min \left[ \text{SOC}(i-1) + \frac{C_A G_d(\beta)}{C_S G_{dm}} \eta_{Bc} - \frac{1}{C_S}; 1 \right] \quad (3)$$

$\eta_{Bc}$  = charge efficiency of the batteries and

$G_d(\beta)$  = total daily irradiation on PV-array.

In these equations, we assume that all energy produced by the PV-array passes through batteries and a constant daily load during whole year [3], [11], [12]. If SOC is greater than 1, it's assumed to equal 1. On the other hand, if a lack of energy occurs, then SOC is negative. This means that the available energy is lower than that demanded by users. Then, the deficit of energy is:

$$E_D(i) = |\text{SOC}(i)| C_S \quad (4)$$

and SOC(i) is reduced to zero. This procedure is carried out for every N days and LLP is given by:

$$\text{LLP} = \frac{\sum_{i=1}^N E_D(i)}{N} \quad (5)$$

Using this procedure, LLP is obtained as a function of  $C_A$  and  $C_S$ . However, we need to calculate  $C_A$  for a determined LLP and  $C_S$ . An interactive method is used to achieve this goal.

The optimisation of the slope of the modules consists of finding the value that leads to the lowest cost of PV-array. Bearing this in mind, the cost of PV-array is normalised to the daily energy demand:

$$P_N = \frac{P}{L} = \frac{C_A}{\eta G_{dm}} P_{mod} \quad (6)$$

where  $P_{mod}$  is the price per square meter of the modules.

The system is simulated for a ten years period and the LLP is obtained from the analysis of the state of charge of the batteries and an initial value of  $C_A$ . An interactive method is used to calculate  $C_A$  for a determined LLP and  $C_S$ .

### 3 PROPOSED ANALYTIC METHOD

The computer program was carried out for 144 Brazilian cities, illustrated in the Figure 1. The system was simulated for a ten years period in order to find the slope of the modules ( $\beta$ ) that leads to the lowest PV-array cost for  $\text{LLP}=10^{-1}$  and  $\text{LLP}=10^{-2}$ . The lowest slope was limited to  $10^\circ$ , because for low latitudes its optimum value is smaller than  $10^\circ$ . Then, the procedure was applied to obtain the isoreliability curves, i.e., the PV-array capacity as a function of the storage capacity, for both LLPs. A isoreliability curve are illustrated in Figure 2.

The isoreliability curves can be fitted by the following equation [2]:

$$C_A = a C_S^{-b} \quad (7)$$

where  $a$  e  $b$  are parameters which depend on the LLP and location. However, analyzing the results we concluded that Equation 7 can be modified by:

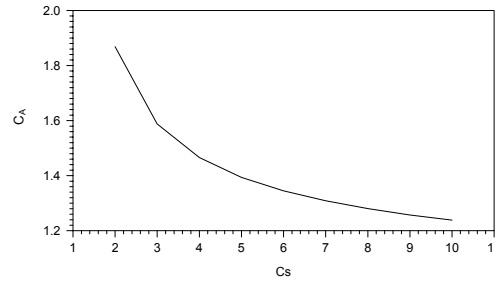
$$\ln(C_A + 1) = a [\ln(C_S)]^{-b} \quad (8)$$

The parameters  $a$  and  $b$  were calculated for the 144 locations and a set of equations was found to estimate both parameters as well as the slope of the modules. Two methods are presented. The method A consists of finding

the parameters  $a$  and  $b$  as a function of the latitude and in the method B these parameters are obtained as a function of the latitude and clearness indexes.



**Figure 1:** Distribution of the 144 Brazilian cities selected to simulate the stand-alone PV system.



**Figure 2:** The isoreliability curves for a stand-alone system at Porto Alegre ( $\phi = -30^\circ$ ) and for  $\text{LLP}=10^{-2}$ .

## 4 RESULTS AND DISCUSSION

We verified that the optimum slope of the modules increases as a function of the latitude ( $\phi$ ), but the behaviour is not linear and depends on the LLP. Then, equations to estimate  $\beta$  as a function of latitude were found, in order to define the method A, for both LLPs. The results are presented in Table I, where  $\phi$  is the absolute of latitude and  $K_{t_{inv}}$  is the daily average clearness index related to June, the month corresponding to the winter solstice.

The comparison of estimated ( $V_{Est}$ ) to simulated ( $V_{Sim}$ ) values is carried out by the average deviation, defined as follows:

$$D_{Aver} = \frac{\sum_{i=1}^n \left| \frac{V_{Est} - V_{Sim}}{V_{Sim}} \right|}{n} 100\% \quad (9)$$

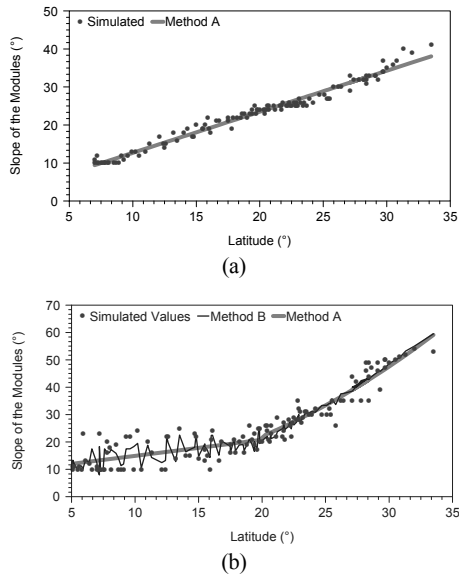
For method A and  $\text{LLP}=10^{-1}$ ,  $D_{Aver} = 4.5\%$ . In this case, the value of the average deviation is low, then it is not necessary to develop method B. However, for  $\text{LLP}=10^{-2}$ , the average deviation, for method A, is equal to 24.6% and 8.4% for latitudes lower than  $20^\circ$  and higher than  $20^\circ$ , respectively. When method B is used this values became equal to 15.9% and 7.9%, for latitudes lower than  $20^\circ$  and higher than  $20^\circ$ , respectively. From an analysis of the influence of the difference between the estimated and

simulated values of  $\beta$ , we concluded that variation of  $10^\circ$  results in an increase cost of the PV-array of 3%.

**Table I:** Equations to obtain the slope of the modules ( $\beta$ ) for  $LLP=10^{-1}$  and  $LLP=10^{-2}$  as well as for both developed method.

| LLP       | Met. | Equations  |
|-----------|------|--|
| $10^{-1}$ | A    | $\phi \leq 7^\circ \Rightarrow \beta = 10^\circ$   |
|           |      | $\phi > 7^\circ \Rightarrow \beta = 1.860 + 1.081\phi$   |
| $10^{-2}$ | A    | $\phi < 5^\circ \Rightarrow \beta = 10^\circ$  |
|           |      | $5^\circ \leq \phi \leq 20^\circ \Rightarrow \beta = 9.067 + 5.865 \times 10^{-1} \phi$                                  |
|           |      | $\phi > 20^\circ \Rightarrow \beta = 5.166 - 2.882 \times 10^{-1} \phi + 5.666 \times 10^{-2} \phi^2$                    |
|           | B    | $\phi < 5^\circ \Rightarrow \beta = 10^\circ$  |
|           |      | $5^\circ \leq \phi \leq 20^\circ \Rightarrow \beta = 38.924 + 5.865 \times 10^{-1} \phi - 54.416 Kt_{inv}$               |
|           |      | $\phi > 20^\circ \Rightarrow \beta = 18.021 - 2.882 \times 10^{-1} \phi + 5.666 \times 10^{-2} \phi^2 - 25.261 Kt_{inv}$ |

Figure 3 compares the simulated values of the slope of the modules with that estimated by method A and method B. We observe that for  $LLP=10^{-1}$ , method A fits in well to simulated data. Similarly, method B yields better fits to low latitudes.



**Figure 3:** Simulated slope of the modules and values estimated by method A and method B for (a)  $LLP=10^{-1}$  and (b)  $LLP=10^{-2}$ .

**Table II:** Equations to estimate parameters  $a$  and  $b$ , for  $LLP=10^{-1}$  and  $LLP=10^{-2}$  as well as for both developed

method. These parameters can be used to size stand-alone systems in Brazil.

| LLP       | Met  | Equations  |
|-----------|--|--|
| $10^{-1}$ | A  | $\phi \leq 20^\circ \Rightarrow a = -2.5690 \times 10^{-3} \phi + 7.6620 \times 10^{-1}$   |
|           |  | $\phi > 20^\circ \Rightarrow a = -7.7868 \times 10^{-3} \phi + 1.4406 \times 10^{-4} \phi^2 + 8.1540 \times 10^{-1}$   |
|           |  | $\phi \leq 20^\circ \Rightarrow b = -2.9121 \times 10^{-3} \phi + 9.2957 \times 10^{-5} \phi^2 + 3.0472 \times 10^{-2}$                                      |
|           | B  | $\phi > 20^\circ \Rightarrow b = 2.0441 \times 10^{-3} \phi - 3.4187 \times 10^{-2}$   |
|           |  | $\phi \leq 20^\circ \Rightarrow b = -8.4906 \times 10^{-1} Kt_{min} + 7.6827 \times 10^{-1} Kt_{min}^2 + 2.3670 \times 10^{-1}$                              |
|           |  | $\phi > 20^\circ \Rightarrow b = -1.5558 Kt_{inv} + 1.3128 Kt_{inv}^2 + 4.6659 \times 10^{-1}$   |
| $10^{-2}$ | A  | $\phi \leq 20^\circ \Rightarrow a = -7.5410 \times 10^{-3} \phi + 1.3774 \times 10^{-4} \phi^2 + 9.4780 \times 10^{-1}$                                      |
|           |  | $\phi > 20^\circ \Rightarrow a = +9.8285 \times 10^{-3} \phi + 6.5120 \times 10^{-1}$  |
|           |  | $\phi \leq 20^\circ \Rightarrow b = 1.7241 \times 10^{-4} \phi + 1.2220 \times 10^{-1}$  |
|           | B  | $\phi > 20^\circ \Rightarrow b = 4.2578 \times 10^{-2} \phi - 6.5387 \times 10^{-4} \phi^2 - 4.6940 \times 10^{-1}$  |
|           |  | $\phi \leq 20^\circ \Rightarrow a = -7.541 \times 10^{-3} \phi + 1.3774 \times 10^{-4} \phi^2 + 4.8608 Kt_{med} - 5.2816 Kt_{med}^2 - 1.4730 \times 10^{-1}$ |
|           |  | $\phi > 20^\circ \Rightarrow a = +9.8285 \times 10^{-3} \phi - 7.2092 Kt_{inv} + 6.5197 Kt_{inv}^2 + 2.6190$   |
| B         | $\phi \leq 20^\circ \Rightarrow b = 3.6277 Kt_{med} - 4.1920 Kt_{med}^2 - 6.2630 \times 10^{-1}$ |  |
|           | $\phi > 20^\circ \Rightarrow b = -5.7709 Kt_{inv} + 4.6904 Kt_{inv}^2 + 1.8933$                  |  |

From Equation 8 and the isoreliability curves determined for each location, the  $a$  and  $b$  parameters were calculated. Then, a set of equations was obtained to estimate these parameters for both LLPs and methods. The results are summarized in Table II, where  $Kt_{min}$  is the

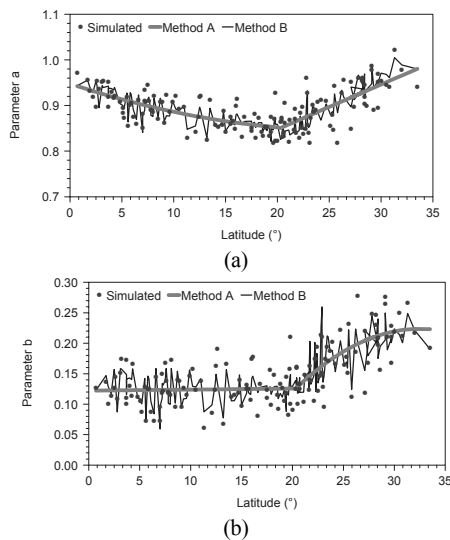
daily average clearness index related to the month that presents the lowest irradiation data.

The analysis of average deviation demonstrates that parameter  $a$  presents good result and the values of  $D_{Aver}$  are lower than 3.3%. For high LLP and method A, the average deviation is of 0.5% and, then, method B is not necessary. From Equation 8, we verify that this parameter influences strongly the value of  $C_A$ , meanwhile, the parameter  $b$  affects the shape of the curve. The average deviation of the parameter  $b$  is high, as shows Table III.

The results obtained with method A and method B as well as the simulated values of parameters  $a$  and  $b$  for  $LLP=10^{-2}$  are presented in Figure 4. We observe that better results were obtained with method B.

**Table III:** Average deviation ( $D_{Aver}$ ) of the parameter  $a$  and  $b$  estimated by using method A and method B as well as for  $LLP=10^{-1}$  and  $LLP=10^{-2}$ .

| LLP       | Met. | Parameter | $D_{Aver}$ (%)       |     |
|-----------|------|-----------|----------------------|-----|
| $10^{-1}$ | A    | a         | $\phi \leq 20^\circ$ | 0.5 |
|           |      |           | $\phi > 20^\circ$    | 0.8 |
|           |      | b         | $\phi \leq 20^\circ$ | 68  |
|           |      |           | $\phi > 20^\circ$    | 53  |
|           | B    | b         | $\phi \leq 20^\circ$ | 38  |
|           |      |           | $\phi > 20^\circ$    | 30  |
| $10^{-2}$ | A    | a         | $\phi \leq 20^\circ$ | 2.4 |
|           |      |           | $\phi > 20^\circ$    | 3.3 |
|           |      | b         | $\phi \leq 20^\circ$ | 20  |
|           |      |           | $\phi > 20^\circ$    | 17  |
|           | B    | a         | $\phi \leq 20^\circ$ | 1.6 |
|           |      |           | $\phi > 20^\circ$    | 2.5 |
|           |      | b         | $\phi \leq 20^\circ$ | 13  |
|           |      |           | $\phi > 20^\circ$    | 12  |



**Figure 4:** Simulated and estimated values of the (a) parameter  $a$  and (b) parameter  $b$ , using both proposed methods and for  $LLP=10^{-2}$ .

## 5 CONCLUSIONS

The method B presents the better results, but if irradiation data does not exist, the method A can be used with only a slightly decreasing of accuracy. Both proposed analytic methods were applied to size a stand-alone system in Torres ( $\phi = -29.3^\circ$ ). This location was selected because the results presented the highest difference between the simulated and estimated values. For  $LLP=10^{-2}$ , this difference is lower than 10% and 6% when method A and method B are used, respectively.

In summary, two analytic methods for sizing stand-alone systems in Brazil were developed and analysed. Such methods can easily be used by PV system designer in order to obtain cost effective systems.

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