PUCRS

ESCOLA DE NEGÓCIOS
PROGRAMA DE PÓS-GRADUAÇÃO EM ECONOMIA DO DESENVOLVIMENTO
DOUTORADO EM ECONOMIA DO DESENVOLVIMENTO

CARLOS MARCHIONATTI

ESSAYS ON DSGE MODELS APPLIED FOR A REGIONAL ECONOMY

Porto Alegre
2024

PÓS-GRADUAÇÃO - *STRICTO SENSU*

Pontifícia Universidade Católica do Rio Grande do Sul

PONTIFICAL CATHOLIC UNIVERSITY OF RIO GRANDE DO SUL

BUSINESS SCHOOL

GRADUATE PROGRAM IN DEVELOPMENT ECONOMICS

CARLOS MARCHIONATTI

ESSAYS ON DSGE MODELS APPLIED FOR A REGIONAL ECONOMY

PORTO ALEGRE

2024

CARLOS MARCHIONATTI

ESSAYS ON DSGE MODELS APPLIED FOR A REGIONAL ECONOMY

Tese apresentada como requisito parcial para a obtenção do grau de Doutor em Economia pelo Programa de Pós-Graduação em Economia da Escola de Negócios da Pontifícia Universidade Católica do Rio Grande do Sul.

Advisor: Professor Dr. Gustavo Inácio de Moraes.

Porto Alegre

2024

Ficha Catalográfica

M317e Marchionatti, Carlos

Essays on DSGE Models applied for a regional economy / Carlos Marchionatti. – 2024.

124.

Tese (Doutorado) – Programa de Pós-Graduação em Economia do Desenvolvimento, PUCRS.

Orientador: Prof. Dr. Gustavo Inácio de Moraes.

1. Regional Macroeconomics. 2. DSGE Models. 3. Rio Grande do Sul. 4. Kansai-RS Model. 5. State Public Debt. I. Moraes, Gustavo Inácio de. II. Título.

Elaborada pelo Sistema de Geração Automática de Ficha Catalográfica da PUCRS
com os dados fornecidos pelo(a) autor(a).
Bibliotecária responsável: Clarissa Jesinska Selbach CRB-10/2051

Carlos Marchionatti

"ESSAYS ON DSGE MODELS APPLIED FOR A REGIONAL ECONOMY"

Tese apresentada como requisito parcial para a obtenção do grau de Doutor em Economia, pelo Programa de Pós-Graduação em Economia da Escola de Negócios da Pontifícia Universidade Católica do Rio Grande do Sul.

Aprovado em 16 de agosto de 2024, pela Banca Examinadora.

BANCA EXAMINADORA:

Prof. Dr. Gustavo Inácio de Moraes

Prof. Dr. Gustavo Inácio de Moraes
Orientador e Presidente da sessão

Prof. Dr. Augusto Mussi Alvim
Coordenador
Programa de Pós-Graduação em Economia

Prof. Dr. Augusto Mussi Alvim

Prof.ª Dr.ª Kellen Fraga da Silva

gov.br Documento assinado digitalmente
ALEXANDRE ALVES PORSSE
Data: 16/08/2024 18:23:54-0300
Verifique em https://validar.iti.gov.br

Prof. Dr. Alexandre Alves Porsse

Prof. Dr. João Ricardo Mendes Gonçalves Costa Filho

To my beloved family and, specially, Vó Helena (Vovózinha).

ACKNOWLEDGEMENTS

From the beginning of 2021 to the end of 2024, this academic study has improved as much as my student capacity allowed. Nonetheless, this was not a solo work: many people came along, and I think that is why it became better than it would have been.

First of all, I would like to acknowledge some professors of Economics from Japan, whose works are the basis of this Thesis: Thank you, Prof. Daisuke Ida and Prof. Mitsuhiro Okano, for responding to several e-mails and providing me with Kansai's DSGE model code. Thank you, Prof. Kenichi Tamegawa, for providing your regional DSGE model codes and useful information.

I also acknowledge the examination board for their suggestions, improvements, ideas, and necessary corrections. Especially the external professors, Prof. João Ricardo Costa Filho and Prof. Alexandre Porsse, who are respectively specialists in DSGE and CGE models.

I would also like to thank some special people who made this work better. Specialists of macroeconomics, DSGE models, mathematics, statistics and econometrics, programming, Latex and academic writing were crucial for the success of this PhD thesis.

I thank Prof. Celso Costa-Junior for his help and support in the Thesis DSGE modeling and Dynare. He is one of the pioneers in teaching this methodology in Brazil, alongside Prof. João Ricardo. Their work is making possible for students from "abroad" Brazilian great centers of macro research to also use this complex and elegant type of method.

A colleague from the Department of Mathematics and Statistics of the Federal University of Rio Grande do Sul, Dr. Eduardo Felipe, who helped me with ideas and math/statistics tasks.

M.Sc. Guilherme Lopes, who helped me with LaTex coding and ABNT norms, and Prof. Felipe Zimman, for the initial template and tips.

My beloved family, who helped me with every step from the beginning to the end of the Thesis: Mother Helena, Carlos Cini, Malu, Wilson, Lou, Vó Helena, Antônio, and Pedro.

Message to all who want to pursue a PhD

The path to completing a PhD. research project is not easy. E-mails, codes, and lessons exchanges are not enough. I certainly think that most doctoral students really appreciate what they are studying and put their best effort into achieving the best possible results. Therefore, even if the work is not highly anticipated, it is the top result of someone's attempt to bring to the world a new finding. Of course, every help in the beginning and middle of the road is very important.

I previously said that e-mail, codes, and lessons exchanges are not enough, and many probably will agree with me. However, one of the first e-mails I sent to Professors in Japan opened doors to this work. That is why I am extremely grateful for Professor Daisuke Ida, author of Kansai Regional DSGE model, who sent the last version of his model and also responded several times to me by e-mail, as well as to Professor Kenichi Tamegawa, for sending his work on a pioneer regional DSGE model that he built.

Alongside technical professionals, many other people helped (technically or not) during the PhD research. My beloved family, my future wife, and some great friends. It is surprisingly, I know, but sometimes the best help comes from a non-technical person thinking in simple terms. Who would have thought that a friend at work, using soccer (Brazilian football) to imagine situations, could help me during complex matters related to dynamic macroeconomics and statistical methods for macro data?

The path to completing a PhD dissertation is not easy, but it is worth it. With faith and perseverance, we can all do great work and extend people's knowledge and imagination!

“Coincidence is God’s way of remaining anonymous.” Albert Einstein

RESUMO

O presente trabalho tem o objetivo de estudar modelos macroeconômicos de metodologia DSGE aplicados para economias regionais, assim como utilizar um modelo DSGE para estudar o Estado do Rio Grande do Sul (RS), localizado no Brasil, e simular a possível trajetória econômica estadual durante o Regime de Recuperação Fiscal (RRF). O primeiro ensaio aborda diferentes trabalhos ao redor do mundo utilizando tal metodologia para economias regionais (subnacionais), monstrando a relevância do tema. No segundo ensaio, um modelo primeiramente desenvolvido para uma região do Japão, chamada Kansai, é aplicado ao Rio Grande do Sul, sendo adaptado ao Estado tanto em sua parametrização, quanto a sua modelagem de equações (ênfase na parte fiscal do governo - divisão dos gastos entre os entes federal e local e inclusão de investimento público). O modelo passa a se chamar Kansai-RS, nos moldes da nomeação do modelo SAMBA, consagrado na literatura brasileira macroeconômica. O modelo, apesar de suas limitações, apresentou resultados interessantes e um ponto de partida para a utilização de tal metodologia para o Rio Grande do Sul. A dinâmica da produtividade na economia do RS se mostrou fundamental para o desenvolvimento do Estado, assim como o uso adequado de recursos públicos por parte do governo local. No terceiro ensaio, simulações utilizando o Kansai-RS foram feitas a respeito do Regime de Recuperação Fiscal, assinado em 2021 pelo RS. As simulações mostraram que o indexador da dívida não é o mais relevante, e sim o crescimento sustentado do PIB estadual. Sem aumento da produtividade do Estado, a dívida se torna recessiva ao RS. Por fim, ressalta-se que a contribuição da tese é sobre revisão metodológica e resultados empíricos.

Palavras-chave: Macroeconomia regional. Modelos DSGE. Rio Grande do Sul. Modelo Kansai-RS. Dívida Pública Estadual.

ABSTRACT

The present work aims to study macroeconomic models applied to regional economies that use the DSGE methodology, as well as using a DSGE model to study the State of Rio Grande do Sul (RS), in Brazil, and also simulating its potential economic trajectory during the Fiscal Recovery Regime (FRR). The first essay addresses different works around the world using this methodology for regional (subnational) economies, demonstrating the relevance of the topic. In the second essay, a model first developed for a region of Japan, called Kansai, is applied to Rio Grande do Sul, being adapted to the State both in its parameterization and in its equation modeling (emphasis on the fiscal part of the government - division of government spending into local and central and inclusion of public investment). The model was named Kansai-RS, inspired by the SAMBA model, which is well-established in Brazilian macroeconomic/DSGE literature. Despite its limitations, the model presented interesting results and provided a starting point for the use of such methodology in Rio Grande do Sul. The dynamics of productivity in the economy of Rio Grande do Sul proved to be fundamental for the state's development, as well as for the adequate utilization of public resources by the local government. In the third test, simulations using Kansai-RS were carried out regarding the Fiscal Recovery Regime, signed in 2021 by RS. The simulations showed that the debt index is not the most relevant factor; rather, it is the sustained growth of the state's GDP. Without an increase in State productivity, the debt becomes recessive for RS. Lastly, we highlight that the present work contributes providing methodological review and empirical results.

Keywords: Regional Macroeconomics. DSGE Models. Rio Grande do Sul. Kansai-RS Model. State Public Debt.

LIST OF FIGURES

Figure 1 – RS Output Cycle Decomposition — 2012-2021 73
Figure 2 – RS Simulated Output and Cycle of Output - 2012-2020 74
Figure 3 – RS Simulated Wages and Cycle of Wages - 2012-2020 75
Figure 4 – RS Simulated Inflation and Cycle of Inflation- 2012-2020 76
Figure 5 – Shock of Local Government Spending to Output, Wages, and Inflation - G^L 78
Figure 6 – Shock of Local Government Investment to Output, Wages, and Inflation - IG^L 78
Figure 7 – Shock of Non-durable Sector productivity to Output, Wages, and Inflation - A^C 79
Figure 8 – Shock of Durable Sector productivity to Output, Wages, and Inflation - A^D 79
Figure 9 – Simulated Interest Rates after a sequence of only local government spending/investment decrease 81
Figure 10 – Nominal Result of RS in Billions of Reais — Prices of 2022 (Deflated by IGP-DI) 88
Figure 11 – RS active and non-active employees expenditure in Billions of Reais -Prices of 2021 (Deflated by IGP-DI) 89
Figure 12 – Debt Cost over State Net Revenue (in %) 90
Figure 13 – Forecast for amortization and interest on RS debt in Billions of Reais — Prices of 2022 (Deflated by IGP-DI forecasts) 91
Figure 14 – Scenario 1 for Output Cycle ($\hat{Y}$): No other shock hitting the economy — 2024 to 2031 94
Figure 15 – Scenario 2 for Output Cycle ($\hat{Y}$): Negative productivity shocks hitting the economy - 2024 to 2031 95
Figure 16 – Scenario 3 for Output Cycle ($\hat{Y}$): Positive productivity shocks - 2024 to 2031 96

Figure 17 – Scenario 4 for Output Cycle ($\hat{Y}$): Negative productivity and increasing interest shocks - 2024 to 2031 97
Figure 18 – Scenario 5 for Output Cycle ($\hat{Y}$): Positive productivity and increasing interest shocks - 2024 to 2031 98
Figure 19 – Output cycle path ($\hat{Y}$) in the presence of tax increases and no other shock hitting the economy - 2024 to 2031 98
Figure 20 – Output cycle path ($\hat{Y}$) in the presence of tax increases and positive productivity shocks hitting the economy - 2024 to 2031 99
Figure 21 – Output cycle path ($\hat{Y}$) in the presence of tax increases and negative productivity shocks hitting the economy - 2024 to 2031 99
Figure 22 – Taylor Rule Estimation for RS - Actual, fitted and residual values for monthly data 119
Figure 23 – RS and Brazil's inflation - Compounded 12 month rate of change . . . 119
Figure 24 – RS Observed Time Series - 2010 to 2021 (in Natural Log) 120
Figure 25 – RS Productivity Series - 2011 to 2021 (in Natural Log)[1] 121

LIST OF TABLES

Table 1 – Summary of Regional DSGE models contributions and results 45
Table 2 – Kansai-RS model parameters (Annual frequency) 63
Table 3 – Kansai-RS model parameters — Fiscal and persistence parameters . . . 64
Table 4 – Shocks to the RS economy during 2012-2022 period 67
Table 5 – Correlations of RS Data 68
Table 6 – Standard-Deviation of RS Data 68
Table 7 – Correlations of Simulated Data: Non-durable productivity shock 69
Table 8 – Standard-Deviation of Simulated Data: Non-durable productivity shock 69
Table 9 – Correlations of Simulated Data: Durable productivity shock 69
Table 10 – Standard-Deviation of Simulated Data: Durable productivity shock . . . 69
Table 11 – Correlations of Simulated Data: Local government shock 70
Table 12 – Standard-Deviation of Simulated Data: Local government shock 71
Table 13 – Correlations of Simulated Data: Central government shock 71
Table 14 – Standard-Deviation of Simulated Data: Central government shock . . . 71
Table 15 – Correlations of Simulated Data: Local and central governments shock . 72
Table 16 – Standard-Deviation of Simulated Data: Local and central government spending shock 72
Table 17 – Variance Decomposition, simulating one shock at a time (in percent) . . 72
Table 18 – Correlations of Simulated Sequence of Shocks 76
Table 19 – Standard-Deviation of Simulated Sequence of Shocks 76
Table 20 – Correlations of Simulated and Observed Series (Not including the Pandemics) 77
Table 21 – Taylor Rule Estimation for RS - Coefficients Results with monthly data 118
Table 22 – Taylor Rule Estimation for RS - Additional Information for monthly . . 118

LIST OF ABBREVIATIONS AND ACRONYMS

RS	Rio Grande do Sul
DSGE	Dynamic Stochastic General Equilibrium
FRR	Fiscal Recovery Regime
IBGE	*Brazilian Institute of Geography and Statistics* - Instituto Brasileiro de Geografia e Estatística
EU	European Union
US	United States (of America)
Sefaz	Secretary of Finance
DEE	Departament of Economics and Statistics
GDP	Gross Domestic Product (Output)

CONTENTS

1 INTRODUCTION . . . 17

2 ON REGIONAL DSGE MODELS: A SURVEY ABOUT THE CURRENT STATE OF THE ART OF REGIONAL DSGE MODELS . . . 19

2.1 INTRODUCTION . . . **19**

2.2 CONTEXTUALIZATION OF DSGE MODELS . . . **22**

2.2.1 Country/Continent DSGE models and beyond . . . 22

2.2.2 Why regional DSGE models? . . . 23

2.2.3 How to Construct a Small Regional Model — Tamegawa's (2013) example 24

2.2.3.1 Tamegawa's (2013) model, parametrization, and features . . . **25**

2.2.3.2 Government block: The division of local and central governments block . **27**

2.2.3.3 Model Results and Lessons . . . **29**

2.2.4 Extended version of Tamegawa's Two-Region model: Multi-Region DSGE model for small regions with local governments . . . 30

2.3 REGIONAL DSGE MODELS FOR SPECIFIC REGIONS . . . **31**

2.3.1 Europe: First attempt to create a two-region DSGE model (Europe and Germany area) and the Greece region example . . . 31

2.3.1.1 Results of the estimation and Home-Foreign Shocks . . . **33**

2.3.2 Multi-Country DSGE Model example . . . 34

2.3.3 Asia: A Regional DSGE Model for Japan . . . 35

2.3.3.1 The construction of specific points of Okano et al. (2015) model for the region of Kansai . . . **36**

2.3.3.2 The main results of Okano et al. (2015) . . . **37**

2.3.4 Asia: A Regional Model for Indonesia . . . 38

2.3.5 Eurasia: A Regional Model for Russia . . . 39

2.3.6 South America: A Regional Model for the Northeast of Brazil . . . 40

2.3.6.1 Ceará's model specifications . . . **40**

2.3.6.2 Paiva's (2019) results . . . **41**

2.3.7 **South America: A Regional Model for the Center-West of Brazil** . . . **42**
2.3.7.1 *Results* **43**
2.3.8 **The regionalization of the SAMBA model** **43**
2.3.9 **A good idea to quantify environmental shocks in regional economies** . **44**
2.4 IMPORTANT NOTES ON REGIONAL DSGE MODELS **46**
2.5 CONCLUSION **47**
3 **KANSAI-RS: A REGIONAL DSGE MODEL APPLIED FOR THE STATE OF RIO GRANDE DO SUL** **49**
3.1 INTRODUCTION **49**
3.2 KANSAI-RS DSGE MODEL **52**
3.2.1 **Households** **52**
3.2.2 **Non-durable goods producers** **55**
3.2.3 **Durable goods producers** **56**
3.2.4 **Calvo Pricing** **56**
3.2.5 **Monetary Policy** **57**
3.2.6 **Government: Important block and adaptations** **58**
3.2.7 **Equilibrium conditions** **59**
3.3 PARAMETRIZATION **59**
3.4 DOES THE KANSAI-RS MODEL EXPLAIN THE RIO GRANDE DO SUL ECONOMY? COMPARISON OF THE MODEL AND DATA MOMENTS AND IRFS **65**
3.4.1 **Moments Comparison: Correlations between observed and model cycles** **67**
3.4.1.1 *Results of productivity shocks: Non-durable* **68**
3.4.1.2 *Results of productivity shocks: Durable* **69**
3.4.1.3 *Results of Local Government Fiscal Shock* **70**
3.4.1.4 *Results of Central Government Fiscal Shock* **70**
3.4.1.5 *Results of Both Governments Fiscal Shock* **71**
3.4.1.6 *Variance and Historical Decomposition of RS Output Cycle* **72**

3.4.2 **Results of Simulated Sequence of Shocks** **73**
3.4.3 **Impulse-Response Functions** **76**
3.5 SENSIBILITY AND IDENTIFICATION ANALYSIS OF KANSAI-RS MODEL **80**
3.6 CONCLUSION **82**
4 **THE FISCAL POLICY OF RIO GRANDE DO SUL: SUBNATIONAL PUBLIC FINANCE ANALYSIS AND SIMULATIONS FOR THE FISCAL RECOVERY REGIME USING KANSAI-RS DSGE MODEL** **84**
4.1 INTRODUCTION **84**
4.2 SUBNATIONAL PUBLIC FINANCE IN BRAZIL **85**
4.2.1 **The State Finances of Rio Grande do Sul** **87**
4.2.2 **Evolution and Negotiation of Rio Grande do Sul Public Debt and the current Fiscal Recovery Regime (FRR)** **90**
4.3 SIMULATIONS FOR THE FISCAL RECOVERY REGIME OF RIO GRANDE DO SUL **92**
4.3.1 **What if the Local Government increases taxes to maintain its spending/investment policies?** **96**
4.4 CONCLUSION **100**
5 **CONCLUDING REMARKS** **103**
BIBLIOGRAPHY **105**
GLOSSARY **112**
APPENDIX A – LOG-LINEARIZED EQUATIONS OF KANSAI-RS DSGE MODEL **113**
APPENDIX B – TAYLOR RULE ESTIMATION FOR KANSAI-RS MODEL **117**
ANNEX A – TIME SERIES OF THE KANSAI-RS MODEL **120**
ANNEX B – UNOBSERVED TIME SERIES THE OF KANSAI-RS MODEL - PRODUCTIVITY DECOMPOSED USING LABOR, CAPITAL AND OUTPUT GROWTHS . . . **121**

1 INTRODUCTION

The present work aims to study macroeconomic models applied to regional economies that use the DSGE methodology. Throughout the decades, DSGE models became more relevant and are now one of the main tools for macroeconomic analysis. The first developed models focused on national economies, such as the USA, countries of the European Zone and Japan. Later, other models have started to analyze subnational economies, i.e., regional economies within a country or larger geographic region, such as a European country. The first essay surveys those DSGE models applied regionally and shows how this type of research is growing in terms of quantity and application around the world, as well as its main characteristics and lessons.

In the second essay, a specific model used in Japan is adapted and applied to the State of Rio Grande do Sul, Brazil, and named Kansai-RS model. The adaptations are for the parameters, which are intended to demonstrate the regional characteristics of RS, as well as for some variables, specially the ones of the government block. For example, central government expenditure enters the output equation and both local and central governments are further expanded, allowing the accumulation of capital stock through government investment. The model, divided into two firms sectors (non-durable goods and services and durable goods and services, the last representing the civil construction sector), two types of households (Ricardians and Non-Ricardians), two fiscal and one monetary authorities show the behavior of the RS economy after fiscal and productivity shocks. The results showed that RS depends on productivity to grow, and the fiscal governments should aim the policies which contribute to this type of growth. Despite its limitations, Kansai-RS model presented good starting results and showed what might improve the modelling of the RS economy and topics for future research.

The third essay used Kansai-RS model to simulate the Fiscal Recovery Regime, an agreement between Brazilian central government and RS local government to start the payment of RS public debt. Basically some part of the local government tax revenues will be drained in order to pay this debt, and then the regional economy might suffer from the lack of public spending and investment. The simulations showed that, although the FRR is significant and drains taxes from local government, productivity shocks are more important for the regional output path. Therefore, it is fundamental that the RS output grows enough, allowing the debt amortization with less harm. On the other hand, since the RS economy is slightly improving in terms of GDP from 2011 on, and have episodes of climatic disasters, such as big droughts, floods and intense rain or warm, there is a warning about the failure in paying the debt installments of negative shocks occur.

Overall, this PhD Thesis, composed by three interdependent essays, explores a new field within DSGE modeling and regional macroeconomics topics to be used for specific regions. Also, despite the limitations of the methodology and Kansai-RS model, it has presented interesting results in order to study the RS economy and its future, as well as identified what drives most its growth (productivity shocks). Moreover, topics for further research, including other types of households, firms and governments modelling, alongside the use of Bayesian estimation on longer time series might improve the analysis and planning of public policies for the State of Rio Grande do Sul.

2 ON REGIONAL DSGE MODELS: A SURVEY ABOUT THE CURRENT STATE OF THE ART OF REGIONAL DSGE MODELS

This essay[1] is a survey about regional macroeconomic models. It will discuss the evolution and main results of one specific methodology named Dynamic Stochastic General Equilibrium (DSGE) models which are applied to regions, sometimes geographically small, sometimes larger (such as entire countries), but always within a larger region and involving and with two types of government (local and central ones) or more. Also, the latest contributions and a summary of these types of models are presented in Table 1.

2.1 INTRODUCTION

In the last decades, in order to address macroeconomic analysis, Dynamic Stochastic General Equilibrium (DSGE) models have been used extensively, especially to verify monetary and fiscal policies outcomes in whole continents or countries. However, when a researcher desired to study a smaller region using a DSGE model, there was a lack of this particular type of methodology applied at the regional level. To overcome this issue, the creation of regional DSGE models was useful to identify particularities of such regions, specially in heterogeneous and large economic spaces such as Brazil, United States (US) and the European Union (EU), for example. Pytlarczyk (2005) was one of the pioneers in creating a two-region model where Germany was the smaller region in relation to the larger one, which corresponded to the EU. Since the model described the heterogeneity between Germany and other EU countries, it could analyze how Germany affected the EU and vice versa, properly.

Later, more models for even smaller open economies/regions came, such as Papageorgiou and Kazanas (2013) analyzing how Greece behaved within the European Union and the global sector. Papageorgiou and Kazanas (2013) built a detailed fiscal policy block, given Greece's high debt levels, making this type of analysis fundamental for the country. Also, Greece does not rely on its own monetary autonomy, because the European Central

[1] The current essay was part of the 13º meeting of the Chilean Society of Regional Studies (SOCHER), which occurred in Valdivia, Chile, from November 29th to December 1st of 2023, at the Austral de Chile University (UACh).

Bank (ECB) conducts monetary policy for most European countries, including Greece. Flores et al. (2016) went further, building a model for multi-country interaction, showing how many regions could interact economically.

Outside Europe, other authors created theoretical and applied models for small-regions in Asia, such as Tamegawa (2013), Tamegawa (2012) and Okano et al. (2015), the last building a model for a specif Japanese region named Kansai, one of the total of nine in Japan. Additionally, a model for Indonesia addressed how monetary policy affected several regions differently within the country (WALUYO, 2017). Another example, alongside the previous models, is the one applied to Russia, built by Dubrovskaya, Shults and Kozonogova (2022).

For South America, more specifically Brazil, three regional DSGE models exist: one focused on the northeast of the country, for the State of Ceará (PAIVA, 2019), simulating changes in the steady state of the region in a closed economy framework; another for the center-west of the country, in the State of Goiás (JUNIOR; TEIXEIRA; SILVA, 2022), addressing fiscal (tax hikes) simulations for the region and the rest of Brazil, in a small open economy framework; and the last one, expanding the national "Stochastic analytical model with a Bayesian approach" (SAMBA) (CASTRO et al., 2015) to incorporate regional features and outcomes, created by Penna et al. (2023).

From different continents around the world, many regional DSGE models came by. It all began with a country being the small region in relation to an entire continent, and then the method was improved for even smaller regions, e.g., states or regions of a country, in an open economy framework or not, and with more than one fiscal authority (usually a local government and a central government). Therefore, the research problems arise naturally by these three questions:

1. Why regional DSGE models, and what is the need of them in macroeconomic analysis?

2. What are their main characteristics and differences in relation to "standard" DSGE

models built for entire countries or continents?

3. What are the main results and lessons presented by the available regional DSGE models?

In order to respond to them, the present survey will follow Christiano, Eichenbaum and Trabandt (2018) methodology, surveying many works, but with the difference of focusing only in regional DSGE models and their applications. Also, the scope of the research does not encompass monetary/currency unions[2].

Therefore, in the following sections we shall analyze those regional models, their particularities and their main objectives, alongside their main results, contributing to a better comprehension of this topic and possible further research developments. This survey can also be used as a guide for lectures.

About the main results of the regional DSGE models, they presented several types of different lessons, from very different fiscal and monetary outcomes to different productivity dynamics too. Since each of them was built for a specific research problem and had particular results, the most appropriate way to present their reviews was to show one by one, highlighting the news and grouping them by some geographic measure, of which the continent level grouping was the chosen one. However, the main results and lessons are also compiled in Table 1 for a quicker and summarized understanding.

After this introduction (Section 2.1), Section 2.2 will present the theoretical basis of a regional DSGE model, which involves the division of government into two or more fiscal authorities, and the presence of the regional economy within a broader framework[3]. This section will respond to why regional DSGE models are fundamental for a good regional macroeconomic understanding, and will detail their broad characteristics. Specifically, one important example, designed for theoretical economies (i.e., applicable to any region), is

[2] For readers interested in the study of regions inside a monetary union using the DSGE methodology, for example, other works such as (MIRKO; SEBASTIAN, 2019) and (MONTEIRO, 2023) bring this type analysis.

[3] Even if it is a closed economy, the central government, representing the national economy, sends financial resources to the local government or applies its own fiscal policy to it. Additionally, the monetary authority, which belongs to the central government, also applies its interest rate affecting the regional/local economy.

the work of Tamegawa (2013). Its modelling for the government block, the model main results, as well as its possible variations will be exposed. Section 2.3 divides the models applied to real regions by area (continents), such as Europe, Asia, and South America. Then, Section 2.4 will highlight important points and best practices when building and calibrating/estimating regional DSGE models. Finally, the conclusion (Section 2.5) shall present the last remarks.

2.2 CONTEXTUALIZATION OF DSGE MODELS

In this section, after some contextualization about the DSGE methodology and its attractiveness, the question about "Why regional DSGE models and the need for them?" will be responded. Later, we shall detail some basic and fundamental concepts about the models applied at the regional level, such as the division between local and central governments (fiscal policy block), using theoretical examples (i.e., not applied for a specific region).

2.2.1 Country/Continent DSGE models and beyond

The state of art in creating macroeconomic models is the one that brings structural relations between all sectors of the economy, from households to enterprises and government, the last one comprising both monetary and fiscal authorities. In line with this characteristic, a Dynamic Stochastic General Equilibrium (DSGE) model can demonstrate the micro-foundations of these relations between all economic agents in a way that the analysis becomes deeper and more accurate, showing the whole behavior between all variables of the model.

After the work of Kydland and Prescott (1982) on Real Business-Cycle (RBC) models in the early 1980s, representing a classical macroeconomic model with full price and wage flexibility, perfect competition, and the dynamics of productivity (real or non-nominal) shocks on the economy, New-Keynesian school of thought features (price and wage rigidity, monopolistic competition, and monetary effects on output level, for example) were incorporated into such models in the 1990s[4] (ROMER, 1993). Alongside these improvements making this macroeconomic methodology more robust and now known as DSGE models, computational power increased significantly from 2000 onwards.[5].

These facts led to a massive quantity of different models trying to represent and analyze different areas around the globe, mostly for the United States and Europe, but also for other countries in different continents. Therefore, hundreds of models emerged world-wide focused on big areas. However, what if there is a need to analyze a small country inside a big continent, such as the European Union, or some even smaller region inside a country, such as a state or a city, using the DSGE methodology?

2.2.2 Why regional DSGE models?

When there was the need to disaggregate from the continent or national level analysis, constructing such models became more challenging, as it requires that the model maintains its stability within the dozens of new equations, parameters, and forward or backward-looking exogenous and endogenous variables for each desired additional region (WALUYO, 2017, page 95). However, analyzing economic policies at national level does not mean it will be accurate for every region, since their structure might differ from each other. One simple question can clarify this point: Shall the impacts of the national monetary policy be the same for every region of a specific country? Probably not. For this type of analysis, Waluyo (2017) created a five-region DSGE model for Indonesia to address this problem. Hence, a researcher or governmental agent who wants to analyze some specific region must pass this point of disaggregation, or the model will not respond adequately to the problems it is meant to address.

In order to surpass the problem of expanding a country/continent model, the first natural step was to create a two-region model (big and small regions interacting with each other). This is exactly what Pytlarczyk (2005) did for a real region (Germany and

[4] Plosser (1989) provides an introduction to RBC models as well as a summary of main details and results for this type of methodology.

[5] DAgostini and Dammski (2022) details in great detail the rise and consolidation of DSGE models, with historical and chronological perspectives. See this work for greater detail about the beginning of such methodology and the most recognized papers and models.

EU) and Tamegawa (2013) did for a theoretical region. Other models will be detailed in Section 3, such as the ones presented in Table 1, where there is “only” one region, but it still depends on the national framework. But, firstly, there are main properties of regional models that should be exposed. For now, we will use Tamegawa (2013) as an example.

2.2.3 How to Construct a Small Regional Model — Tamegawa’s (2013) example

The literature to construct a regional DSGE model, wherever the region is on the globe, is quite reasonable. Tamegawa (2013) shows that it is possible to create a regional model that describes a prefecture in Japan, some state in the US, or a country in the UK, for example, in contrast to most of the macro-regions DSGE models made for big countries or an entire continent such as Europe. The author highlights that there are small regions whose output is a small fraction of the nation/continent GDP, even smaller than 1% of the total production of goods and services in the whole area. In this context, “small region’s policymakers need an effective small-region DSGE model to evaluate their policies”, as Tamegawa (2013, page 1) argues in his study, and what can be the main objective of other regional models.

The main goal of Tamegawa (2013) is to create a DSGE model in which the local economy, ruled by two types of fiscal authorities (local and central), does not affect the national economy, since “the region’s activity does not alter both interest rates [. . .] and prices” (TAMEGAWA, 2013, page 1). This perspective also creates the possibility of a non-crowding-out effect, since the increase in the local government expenditure shall not increase the nation’s demand for money and hence interest rates, which would lead to a decrease in private investment and consumption. Thus, this is one new possibility that Tamegawa (2013) brings to this type of research.

Going further, Tamegawa (2013) shows that the small-region DSGE model is similar to a small-open economy DSGE model (in the line with Papageorgiou and Kazanas (2013)), without, however, the international trilemma, in which monetary policy must not affect the economy that operates with a fixed exchange rate. Here, this is not important for the small-region, and the basic framework is the following: the national fiscal and monetary policies can affect the regional economy, as well as the local government can alter (only) economic activity of its own area. Then, many types of economic policies coming from both governments can alter economic variables of the region, depending on the parametrization as well. Let us now examine specific points of this theoretical model.

2.2.3.1 Tamegawa's (2013) model, parametrization, and features

The model of Tamegawa (2013) assumes 4 agents: households, wholesale goods producers, retail goods producers and the government, divided by fiscal and monetary authorities. Also, their behavior is given by macroeconomic states, such as GDP and nominal interest rate. After detailing the usual way households and firms maximize their utility and profit, the author focus on the government block.

Here, we have the most important difference from the standard (nation or continent-wide) DSGE models, which is the division of the government in relation to fiscal policy. Also, there are different possibilities for debt modelling: only central government can issue bonds; only local government can issue bonds; both can; or, finally, neither can. Tamegawa (2013) chose to only the local government to issue bonds, while Okano et al. (2015) chose to only the central government issue bonds. Since every region is different, the choice about which type(s) of government(s) may issue bonds is significant for the model outcomes. Moreover, this model feature is also important when analyzing the smaller region because the results can differ substantially from a local government that issue bonds to a local government without the possibility to issue bonds. In the presence Ricardian and Non-Ricardian agents in the economy, the transmission of shocks and other variables dynamics may differ[6]. Tamegawa (2013, page 5) explains this point:

> "In our settings, the central government's spending and the local government's spending have an equivalent effect on output because local debt does not affect a small region's economy other than through the local tax and local bond rates. However, if non-Ricardian households exist

[6] In the DSGE literature, the model may bring two types of households: one who can borrow money in the financial market and another who spends every earning in the same period which it is received. In the same line, if a government must not borrow (issue bonds to conduct fiscal policy when deficits occur), it could be called a non-Ricardian as well, since its budget can not be extended to receive resources from future periods. Cavalcanti and Vereda (2015) brings a detailed model with this perspective for households, for example.

> as in Galí, López-Salido and Vallés (2007) and central debt and local debt have a different risk premium, a difference emerges. The increase in government's spending raises the bond rate. This leads to a higher tax rate, and therefore, if governments have different elasticity of tax with respect to bonds, it yields the different consequences".

About the parameters, Tamegawa (2013, page 5) states they should be estimated to match actual economic data of the region, which is reasonable for most models when there is public data about the macroeconomic variables of the regional agents. For example, the author states that consumption share to GDP $\left(\frac{c}{y}\right)$ is around 56%, while governmental expenditure share $\left(\frac{g}{y}\right)$ is around 20%. Many "national" parameters can be imported from other large region models, as well from international literature. However, there is a characteristic that must be detailed for open economy regional models: since the policymakers are interested in the effects of local fiscal shocks in Tamegawa's model (TAMEGAWA, 2013), there is one parameter that modifies completely their behavior, which Tamegawa (2013) states to be the imports elasticity. When there is an increase in demand due to local fiscal policy, and interest rates remain unchanged (no crowding-out effect), depending on the imports elasticity, there will be more demand from goods and services from "abroad" (other states or regions) or not. In the second case, this demand will boom more local economic activity because local sales would increase more than imports. So, imports elasticity is important for this type of analysis and for the model construction and parametrization if it is an open economy.

For a researcher who desires to build a local DSGE model, Tamegawa's methodology (TAMEGAWA, 2013) helps to construct (build the equations) the model, learn how to search for ideal parameters, initiate the assumptions for it, run simulations and compare with other models. Notice that there might be a few mandatory differences when constructing a new model, such as the local government being able or not to issue bonds, and other possible differences, such as an open or closed economy framework and other common DSGE features, like household behavior and private investment (firms) technologies and preferences. Hence, Tamegawa (2013) is a good theoretical model starting point if one researcher wants to build their own applied model for some specific region.

2.2.3.2 Government block: The division of local and central governments block

About specific equations of Tamegawa (2013), the ones which turned it regional (by including a local and a federal government) are crucial. While monetary policy follows the standard DSGE model results, fiscal policy shocks have different outcomes, since both local and central governments affect only the region's economy. Also, there is no crowding-out effect, because interest rates do not rise after a fiscal shock.

The new and important government block creates two types of government entities: local and central. Also, we highlight that it is possible to create a third government entity or more, such as a municipal government, according to the research problem. For the local government equation, Tamegawa (2013) assumes that it spends G_t^L and collects a lump-sum tax T_t, and, if it is insufficient to finance the governmental spending, it issues bonds (b_t) with a gross nominal interest rate of R_t^b. Note here that we have a difference from other regional models, since, for example, the federations units of Brazil (the states) are not allowed to issue government bonds for debt funding (PAIVA, 2019) (JUNIOR; TEIXEIRA; SILVA, 2022). So, for their modelling, local government is intertemporally constrained.

Tamegawa's local government has the following budget constraint:

$$b_{t+1} = \frac{R_{t-1}^b}{(P_t/P_{t-1})} b_t + (G_t^L - T_t). \tag{1}$$

Also, Tamegawa assumes that local government bond rate is related to its deposit rates as follows:

$$\log R_t^b = const. + logR_t^N + \rho_R \log b_{t+1}, \tag{2}$$

where $\rho_R \log b_{t+1}$ is a risk premium, the constant is the value that is consistent with the steady state value and R_t^N is the national interest rate, set up by the central government monetary authority.

For the local government budget constraint, there is the need to set it in such way that it maintains its sustainability in the long run, which means that the local government can not issue debt and increase the debt level permanently. With this assumption, the tax rule is set as follows:

$$\log T_t = constant + \rho_{tax} \log b_t. \tag{3}$$

Local government expenditure is set in the following manner:

$$\log G_t^L = constant + \rho_{G,L} \log G_{t1}^L + \epsilon_t^L. \tag{4}$$

Finally, “the central government can consume the small region’s goods denoted by G_t^C and this is defined as follows” (TAMEGAWA, 2013, page 3):

$$\log G_t^C = constant + \rho_{G,C} \log G_{t-1}^C + \epsilon_t^C. \tag{5}$$

If the researcher wants to simulate the effects of national level shocks on the small region’s economy, and vice versa, it is necessary to construct a macroeconomic block. According to Tamegawa, his model endogenizes output, interest, and inflation variables with a simple dynamic IS-LM model expressed in three equations. One for an IS curve (Euler equation), dynamic LM curve (the Taylor Rule) and a New-Keynesian Philips Curve. Tamegawa (2013) highlights that it is also possible to use “a full-blown DSGE model such as Christiano, Eichenbaum and Evans (2005)”. The macroeconomic log-linearized block is as follows:

$$E_t[\hat{Y}_{t+1}] + [\hat{\pi}_{t+1}] = \hat{Y}_t + \hat{R}_t^n. \tag{6}$$

$$\hat{\pi}_t = \beta E_t[\hat{\pi}_{t+1}] + \kappa \hat{Y}_t + \epsilon_t^s. \tag{7}$$

$$\hat{R}_t^n = \psi \hat{\pi}_t + \epsilon_t^R. \tag{8}$$

where Y_t, with capital Y, is the national output, and inflation (π_t) and nominal interest rates (R_t^n) are the other two exogenous variables that were made endogenous.

With the equilibrium condition (equation 9), it is possible to simulate local and central expenditure shocks, monetary policy shocks, the behavior of interest and inflation, the local debt dynamics, how net exports behave and how households and firms are affected by those policies, as examples.

$$y_t = c_t + i_t + g_t + nx_t, \tag{9}$$

where $g_t = G_t^L + G_t^C$. Remembering that G_t^C is central government expenditure “inside” the small-region. Also, equation 9 has two more aggregations: $c_t^{rr} + c_t^s = c_t$ is the total consumption of the region, where “rr” stands for remaining regions consumption and “s” stands for small-region consumption. The same is applied for investment: $i_t^{rr} + i_t^s = i_t$. Net-exports is the exports minus imports of the region.

2.2.3.3 Model Results and Lessons

Tamegawa (2013, sections 4 and 5) shows the impulse response of fiscal, monetary, and productivity shocks, and how local and national outputs are differently affected. For fiscal shocks, while a local government expenditure shock boosts local output, national output, interest, and inflation remain constant. Central government expenditure shock presented similar dynamics, since non-Ricardians are not present in this model (according to Tamegawa’s reference 2.2.3.1). Another important point is the possibility of a crowding-in effect: after a government expenditure shock, interest rates remains constant and consumption does not change either. Then, Tamegawa (2013) states that “the fiscal multiplier is larger than 1 because the countercyclical markup boosts labor demand and investment demand. Thus, government spending has crowding-in effects on investment. This property is not obtained in standard DSGE models.”.

Local and national economies have different dynamics when a monetary shock happens, as shown in Figure 4 (TAMEGAWA, 2013, page 7), since national output increases more than the regional output. For technology and productivity shocks, there are differences too. While a regional technology shock (in the Cobb-Douglas function of firms, represented by A_t) boost the regional economy, national output is not affected at all. On the other hand, a TFP shock (total factor productivity), which is a ϵ_t^s, affects more the national than the regional economy. Tamegawa (2013) also highlights that a technology shock declines the regional output, but increases it later. The explanation is that wholesale prices decrease, leading to a decrease in labor demand. However, technological advances raise the marginal output of capital and increase investment since the value of capital has risen.

About the lessons of Tamegawa (2013), the main ones are on how to model agents and firms (allowing heterogeneity or not), the government's budget constraints (allowing debt or not), if it will be a closed or an open economy (interaction between the region and other regions), and the parameters. While the news are for the division of the government block, every feature shall modify the results that the model will present, as explained previously[7].

One last point about the government block is how the taxes and fiscal rules will be modelled. While Tamegawa (2013) used a simple rule for local government debt stability, the way taxes are split between governments and other types of fiscal rules might modify the model outcomes as well. Okano et al. (2015) shows a way to split the collected taxes, for example.

2.2.4 Extended version of Tamegawa's Two-Region model: Multi-Region DSGE model for small regions with local governments

After the presentation of Tamegawa (2013), the same author published earlier, in 2012, an extended regional DSGE model. This regional DSGE model actually presents the possibility of a multi-region DSGE, which is set for two or more regions, and the existence of more than one local government. Tamegawa (2012) incorporates others characteristics for households (Non-Ricardian and Ricardian) and private and public investment. However, the main assumption about the existence of local and central governments is still present in the model, as well as expanded: now we have two local governments and one central government. Tamegawa (2012) shows the difference between four types of fiscal simulations: where the first local government implements a fiscal expansion, the second local government implements a fiscal expansion, the central government implements a fiscal expansion in the region of the first local government, or in the region of the second local government.

Differently from an entire country, the effects of a certain fiscal policy are not uniform since there are regional heterogeneities, and the fiscal multipliers (parameters that

[7] Tamegawa (2013) divides consumption and investment into regional and remaining regions ones, which can be stated as innovation as well.

measure the impact of fiscal stimuli in the model) can alter significantly depending on which region and what government (local or central) implemented it. In Tamegawa's words (TAMEGAWA, 2012), "Our main result is that the magnitude of the fiscal multiplier depends on which government implements the fiscal policy and where the policy is implemented. This implies that to measure fiscal multipliers, we have to consider heterogeneity across regions". Hence, a difference between a state fiscal policy may have not equal impacts such as a national fiscal policy, as shown earlier in 2.2.3.3.

The idea of Tamegawa is pretty similar to Lindé (2018) when this author posits the possibility of a DSGE model composed by a core region and its satellites, which may be an advance for this type of research. Lindé (2018) uses an example of core euro area economies, such Germany and France, and the peripheral euro economies (Italy and Spanish). A shock may have different impacts in aggregate terms and in relation to the peripheral area only (LINDÉ, 2018, page 282)[8].

2.3 REGIONAL DSGE MODELS FOR SPECIFIC REGIONS

In this section, an introduction to the available and up-to-date regional DSGE models shall be presented. After the presentation of such models, grouped by their respective areas (continental location) they are located, a summary of the ideas, lessons and main results of these regional models is presented in the Table 1.

2.3.1 Europe: First attempt to create a two-region DSGE model (Europe and Germany area) and the Greece region example

The paper of Pytlarczyk (2005) offers a valuable perspective of a regional DSGE Model for Germany, a small open economy without its own monetary authority, as the monetary authority in Europe is the European Central Bank (ECB). Similarly, the paper by Papageorgiou and Kazanas (2013) brings another perspective as well in this context for

[8] See the example at the end of page 281 and the beginning of page 282. Lindé explains that a European Central Bank policy could have different outcomes in terms of the whole Euro area and in terms of the smaller regional economies. This is exactly what Tamegawa suggests and most authors in Table 1 apply in their research, using a national economy and a small region.

Greece (a small open economy without monetary authority and with a high debt level relative to its GDP). Apparently, Pytlarczyk (2005) was the pioneer in dividing a DSGE model into more than one region.

Pytlarczyk (2005) starts by pointing that "the vast majority of existing DSGE models for the euro area treats this conglomerate as a single country" (PYTLARCZYK, 2005, page 2). Hence, the main focus of this paper is to disaggregate the homogeneous single-area of Europe into a two-region model. According to Pytlarczyk (2005), this approach is valid because there is significant evidence of structural heterogeneity between Germany and the rest of the European Monetary Union (EMU).

According to Pytlarczyk (2005), the model is an extended version of both the Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2001) models, incorporating specific elements related to the currency union into his own model. Another interesting point is the one which Pytlarczyk (2005) identifies distinct permanent technology shocks for each region of the model (Germany and other countries of Europe) and they are assumed to be cointegrated[9,10].

Pytlarczyk (2005) also examines how a single currency would affect the transmission of shocks in the model, and the results are satisfactory. However, the author claims that for each monetary regime (flexible exchange rate or monetary union settings), the results may vary. Therefore, a DSGE model for each type of monetary setting is necessary to analyze the different types of results. In short, the estimated DSGE model of Pytlarzcyzk (2005) replicates with significance cross-country correlations of real and nominal variables and finds international spillover effects between them, which are attractive results. Also, the author uses both German data (Volkswirtschaftliche Gesamtrechnung (VGR) database)

[9] Because of this feature, Pytlarczyk (2005) highlights the model is able to match low and high frequencies of the data, integrating growth and business cycle theory (long and short-run macroeconomic conditions).

[10] Cointegration is a property of time series data which demonstrates that two or more historical time series can have a long-run relationship, although their short-run changes may vary. This means that the econometricians can estimate a more sophisticated model including both short and long-run characteristics of the series and create a model with more information, since the series can be estimated in levels, differently from non-cointegrated series, which need to me estimated in stationary (differentiated once or twice if not stationary) condition. For more information about cointegration, see Enders (2015).

and the whole aggregated data of the European Union (Area Wide Model (AWM) database). This allows the model to take advantage of the disaggregated information of the analyzed region. Lastly, the author used Bayesian techniques to estimate some parameters and calibration for others. Pytlarczyk (2005) highlights that using prior information for the parameter estimation to enrich the model and other calibrated parameters are the simple mean of the respective data series from the first quarter of 1980 to the last one of 2003 (PYTLARCZYK, 2005, page 28).

2.3.1.1 Results of the estimation and Home-Foreign Shocks

In section 6 of Pytlarczyk (2005), the author shows the results of parameter estimation, impulse response functions (IRF) and unconditional second moments of the model. Also, he highlights that even small variations in the model structure and on the estimation data substantively change the results of the analyzed DSGE model. The author begins estimating the baseline two-region model "with variable capital stock and integrated good and financial markets" using two types of data sets:

1. Estimation of the model in log-differences of the raw data.
2. Using the HP-filter to smooth and find the long-run trend of the time series.

Some results of the estimates indicate that, in the long-run, the response of nominal interest rate to inflation is greater than one (1.59), which follows the basic Taylor Rule. Another interesting fact is that wage stickiness in Germany is lower than in the rest of the European Monetary Union (EMU). Tables 4 to 11 show the whole set of prior and posterior estimates of the parameters that model calculated with the available data and this study also indicates a fine adjustment to the data, according to Pytlarczyk (2005, pages 36 and 37).

Other interesting results come from the IRF, showing how home and foreign shocks impact both economies, dividing the shocks into flexible exchange rate or monetary currency union. As we can see, in this example of an investment shock hitting the German economy, GDP increases in both Germany and Europe. However, employment falls in Germany, remaining stable in Europe. Consumption and investment increase in Germany, but remain stable in Europe.

Here, the main contribution discovered, while creating a regional model, was comparing IRF from both "home" (small region) and the "foreign" economies: the foreign economy (EU), using monetary shocks, affected Germany and the Euro-Zone differently. Another point that Pytlarczyk (2005) contributes is the use of data from both home and foreign economies, i.e., the smaller and the bigger regions. Usually, both regional/local and federal/central governments around the world have institutes and secretaries which have economic data and may be used for this kind of research proposal.

Another regional model, inside Europe (PAPAGEORGIOU; KAZANAS, 2013), presented key contributions as well. Firstly, since Greece is a highly indebted country, facing public finance problems for years, it required fiscal measures such as government expenditure cuts and income taxes rises (PAPAGEORGIOU; KAZANAS, 2013, page 10). Therefore, this led the research to analyze a region in where the local government had a weak fiscal condition. The main results showed that the best option to improve fiscal and external imbalances is to reduce government spending via cuts in public sector wages, government transfers and public sector employment. On the other hand, the most harmful way in terms of output losses is to increase taxes on labor income (PAPAGEORGIOU; KAZANAS, 2013, pages 38 to 46).

2.3.2 Multi-Country DSGE Model example

The paper of Flores et al. (2016), about a multi-country DSGE model, shows the possibility of creating a model in terms of many (big or not) geographical units (in this case, countries). He addresses the first two big questions beyond a two-region model: What if there are many (multi) regions, and how do they interact with each other? How could we distinguish the specific impacts of one region on many other members of a multi-region model? Flores et al. (2016) explains further that there is a gap in recent literature, stating that there was only a two or three-region DSGE model, usually composed of the United

States, Japan or the Euro-zone as a whole, or a model with a monetary authority such as Europe with the European Central Bank (ECB), which is exactly what Pytlarczyk (2005) did for Germany and the rest of Europe (a two-region model with a common monetary authority).

For a multi-region analysis, Flores et al. (2016) shows how to build a DSGE model that specifies different regions around the globe (OECD, Euro-Zone and NAFTA countries), and, moreover, how to analyze the impact of these regions on each other. There are many possibilities of interactions (shocks) that shall be done following what Flores et al. (2016) showed, such as global and regional supply and demand shocks. The results varied according to the parameter specification and the region that created and was affected by the shock. For example, Canada and Mexico presented different sensitivities for US monetary shocks.

2.3.3 Asia: A Regional DSGE Model for Japan

Now, analyzing real regional models in Asia, in this case Kansai, Japan, Okano et al. (2015)[11] bring the same perspective as Tamegawa (2013) for constructing a regional DSGE model using both local and central government. Okano et al. (2015) also explains how a researcher could benefit from a regional model:

- Specific simulations for the region shall reveal results that a national DSGE cannot.
- One of the most interesting details is the following: the regional differences can be expressed with different parameter calibration. One set of calibrated parameters for Kansai (or some state of a country) and another set for another Japanese region (for example, Kanto) or country state. The researcher can use local data to calibrate the parameters. This leads to discovering “differences between their underlying economic structures. . . ” (OKANO et al., 2015, page 4).
- The relation between local and national economies may facilitate testing scenarios (what should happen in the small region if the federal government expands its fiscal

[11] In the subsection of South American models (in Brazil), two other authors apply some of Okano’s characteristics (OKANO et al., 2015) for Brazilian states, which are Paiva (2019) and Junior, Teixeira and Silva (2022), demonstrating that are plenty of possibilities for this kind of macroeconomic research.

policy or alters tax allocations?). In Okano's words: "For example, by identifying the fiscal structure of a region and that of the central government, we can run simulations wherein policy alters links between the two governments (e.g., tax allocations)." (OKANO et al., 2015, page 5).

Hence, a regional model for Kansai could show differences among regions inside Japan. Okano et al. (2015) also showed significant improvements in relation to Tamegawa (2013), where the governments split taxes and the central government transfers resources to the local one.

2.3.3.1 The construction of specific points of Okano et al. (2015) model for the region of Kansai

One important point of Okano et al. (2015)[12] is the way the authors differentiate the local and the central governments. Here, the central government issues bonds instead of the local government, as modeled in Tamegawa (2013). Also, there is a difference in receiving the collected taxes over households and firms between the central government and the local one, as well as there is a specific way which the central government passes some collected taxes to the region of Kansai. Let us take a look at the government (G and T) equation of expenditure and revenues (τ) for the local government budget (the superscript L is for the regional government while the superscript C is for the central one):

$$
\begin{aligned}
P_t G_t^L = (1-\theta_C)(\tau_t^C P_t^C C_t + \tau_t^D P_t^D I_t^D) \\
+ (1-\theta_W)(\tau_t^{WC} W_t^C N_t^C + \tau_t^{WD} W_t^D N_t^D) \\
+ (1-\theta_F)\tau^F(\phi_t^C + \phi_t^D) + \omega T R_t. \quad (10)
\end{aligned}
$$

The first line of equation 10 represents the taxes over consumption of non-durable (C) and durable (D) goods, while the second represents taxes over wages from work producing non-durable and durable goods and the last one over firm's profits from non-durable and durable goods. The last component ωTR_t represents the amount of taxes

[12] In the subsection of South American models (in Brazil), two other authors apply some of Okano et al. (2015) characteristics for Brazilian states, which are Paiva (2019) and Junior, Teixeira and Silva (2022), demonstrating that are plenty of possibilities for this kind of macroeconomic research.

that is transferred from central to local government. Now, turning to the central (federal) government, the equation is exactly the same, increasing only the fact that the national budget can issue debt in terms of bonds (B):

$$
\begin{aligned}
P_tG_t^C - B_{t+1} + R_tB_t + \omega TR_t = \theta_C(\tau_t^C P_t^C C_t + \tau_t^D P_t^D I_t^D) \\
+ \theta_W(\tau_t^{WC} W_t^C N_t^C + \tau_t^{WD} W_t^D N_t^D) \\
+ \theta_F \tau^F(\phi_t^C + \phi_t^D) + T_t. \quad (11)
\end{aligned}
$$

Note that along the fact of issuing debt the federal government receives taxes in the opposite proportion of the local government. While one receives θ_C, the other receives $(1-\theta_C)$. Also, T_t stands for resources used for the central government policy but not collected inside the local economy.

2.3.3.2 The main results of Okano et al. (2015)

Okano et al. (2015) runs several impulse-response functions to simulate different shocks for this specific region in Japan and to compare them with a neighboring region, named Kanto. In this perspective, there are different outcomes for productivity shocks on non-durable goods (positive and negative ones) and tax shocks (tax hike in consumption). Both central and local governments policies can affect the local economy differently and create several scenarios for analysis.

For productivity shocks on non-durable, when it was positive, investment, and output increased. However, the bigger output suppresses inflation and consumption increases. The declining inflation rate makes real interest higher, which negatively affects inventory investment (Tobin's Q). Even though investment in non-durable goods increases, housing investment and consumption decline (since real interest rates are higher).

On the other hand, a negative shock on the productivity of non-durable goods has other results. First, this type of shock decreases investment in non-durable goods, lowering both consumption and output in Kansai. Since Kansai has a higher adjustment cost for investment, it suffers way more than an average region would. According to Okano et al. (2015, page 15), if Kansai had an adjustment cost close to the Japanese average, the investment adjustment would be instantaneous, while in Kansai it takes more periods.

Later, a negative productivity shock to non-durable goods hits the economy again. However, the focus now is on the adjustment cost in housing investment (investment in durable goods). Okano et al. (2015) uses two different values for housing investment adjustment cost in the simulations, and the simulation using Kansai higher adjustment cost shows a decrease in investment on non-durable goods and a tiny increase in housing investment and demand. In the overall of Japan, housing investment and demand are higher when a negative productivity on non-durable hits the economy. This difference in residential investment adjustment cost "emerges as one cause for stagnation in Kansai's economy" (OKANO et al., 2015, page 16).

The tax shocks simulations are even more harmful for Kansai. Two announced increases in non-durable goods taxes, in period 5 and 10, show interesting results. The population starts demanding more of this type of good, but in a decreasing rate until period 5 is reached (last-minute demand). After period 5, household consumption is relatively restored until period 10, but does not return to the consumption level of period 5 or earlier. After period 10 (second tax increase), Kansai consumption drops even more and does not reach its steady state value in the twentieth simulation period.

2.3.4 Asia: A Regional Model for Indonesia

For another application in the context of an Asian economy, Waluyo (2017) creates a two-region DSGE model applied for five different regions in Indonesia (national economy and the smaller region). The main goal is to address the following question: "Is the Central Bank of Indonesia implementing optimal monetary policy for all five regions? Do the results differ when considering the five regions separately as opposed to the whole nation?". So, the research problems are now focused on regional monetary policy outcomes. The model presents the following blocks:

- A household block;

- A firm block;
- A terms of trade block;
- A national risk sharing block;
- Price setting block;
- Market clearing block;
- Monetary Authority block;
- Fiscal policy block.

After the construction of the model, Waluyo (2017) estimates the parameters for all five regions, and then applies an interesting exercise to quantify if the loss function of output gap and inflation gap is minimized for each region. It is found that there must be a better monetary policy rule in order to achieve the best outcome for each region, and observing the whole country may induce a public planner to not reach the optimal results using monetary policy.

2.3.5 Eurasia: A Regional Model for Russia

The paper of Dubrovskaya, Shults and Kozonogova (2022) creates a regional DSGE model for Russia, a country located in both Europe and Asia. Dubrovskaya, Shults and Kozonogova (2022, page 9) highlights that Russia is divided into 78 regions, hence the need to disaggregate the national analysis becomes even more important. Also, it is interesting to highlight that Russia is the biggest country in territory dimension in the world. Therefore, the regional analysis of this country becomes more necessary and challenging as well.

Dubrovskaya, Shults and Kozonogova (2022) gives more importance to regional fiscal policy which increases human capital, e.g., healthcare and education. The authors posit that each Russian unit (or region) use their local budget constraint in a long-run balance, so revenues and expenditures are equal, and the regions divide such expenditures into public obligations and human capital development. After the model is built and log-linearize by Uhlig's method (UHLIG, 1999), Dubrovskaya, Shults and Kozonogova (2022) made the parametrization using Russian data and then ran simulations for shocks to the endogenous variables within the horizon of 20 periods (years) ahead. We shall see their results now.

Dubrovskaya, Shults and Kozonogova (2022) results showed that, to achieve short-run goals, economic policy should focus on current budget expenditure. On the other hand, spending on education and healthcare creates an impact on the stock of human capital and long-run economic growth. Also, it is important to highlight that, according to Dubrovskaya, Shults and Kozonogova (2022), investment effects on human capital can be underestimated if expressed in variable deviations. Investment effects may shift the path of potential growth to a higher level as well.

For countries with dozens of regions, the interaction becomes even more intense and deeper. Inter-regional flow of goods, labor, and capital can change the results of the model, then even more different calibration values must be tested. The way Dubrovskaya, Shults and Kozonogova (2022) showed attractive results for only two regions interacting, positing that more sophisticated outcomes can appear when analyzing more than two small regions.

2.3.6 South America: A Regional Model for the Northeast of Brazil

In the Northeast of Brazil, a regional DSGE model was built for a specific state. Paiva (2019) builds a model similar to Okano et al. (2015). Basically, it is a regional model for the State of Ceará, with a local government and the federal government interacting with the smaller region. It includes households' behavior and private and governmental investments. The economy is not open, however, which means there is no interaction with others states in terms of imports and exports. Let us take a look at this DSGE specifications.

2.3.6.1 Ceará's model specifications

Firstly, after the DSGE model was mathematical built, the calibration of the parameters showed to be definitely important as well, as the values can change significantly in the results. Otherwise, the model would suggest worse impulse responses and sensitivity analysis. Paiva (2019) brings the GDP value for the year of 2019, as 126.05 billion Reais.

Consumption and private investments were defined by their share to GDP. Additionally, Paiva (2019) shows that government expenditure C_{g_t} represents the spending with employees and other current expenses. On the other hand, public investment I_{g_t} represents the total amount of capital that is used to increase the potential production of the economy through infrastructure (direct way) or investing in the health and educational systems (indirect).

Regarding the configuration of the system of equations, there are nine endogenous variables and two exogenous ones. $Y_t, C_t, I_t, C_{g_t}, I_{g_t}, TR_t, K_t, T_t, L_t$ are the endogenous, while $TR_t(FG)$ and TT are the exogenous variables. They represent, respectively, output (GDP), private consumption, private investment, government expenditure, government investment, social transfers for local families, stock of capital, state taxes and labor supply. For the exogenous ones, they are transfers from Federal Government to the local government and state taxes over local families.

About the parameters themselves, Paiva (2019) classifies them into 3 types: fiscal parameters, structural parameters and behavioral parameters. The first type refers to collected taxes and public spending. The second type consists of technological parameters such as depreciation and total factor productivity. Finally, there are the behavioral parameters that represent the preferences and intertemporal discount factor of the families.

2.3.6.2 Paiva's (2019) results

Paiva (2019) focuses on two type of simulations to identify how effective and sustainable the fiscal policy is for the regional (local) economy: the first one attempts to answer this question by running permanent shocks on public investment, in terms of increases in the physical capital stock. The second one focuses on temporary shocks to public investment, which are later replaced by increases in public consumption.

The first scenario shows outstanding results because of the increase in public investment. Long-run GDP grows by almost 2%, while consumption, public investment and even private investment grow as well (1.92%, 10.86% and 1.72%, respectively) (PAIVA, 2019, page 40). Basically the greater public investment increases the public stock of capital, growing output, and the greater labor supply also favors the economic development of the region. As total consumption falls (both public and private), leisure becomes more costly and the households decide to offer more of their workforce to maintain their utility levels. Regarding the private stock of capital and its financial return, as the private stock decreases, its productivity increases, thereby rising the profitability for the entrepreneurs. Lastly, local revenues (State of Ceará) also increase, based on the improved results of the other main endogenous variables (GDP, consumption, and investment).

On the other hand, temporary shocks on public investment create a totally different scenario. Every variable, except for public consumption, falls from its previous steady state (PAIVA, 2019, page 45, Table 2). For example, GPD falls by about 5% and public capital stock by falls about 26%. Therefore, those two types of fiscal simulations bring interesting results for researchers interested in investigating deeper regional fiscal shocks.

2.3.7 South America: A Regional Model for the Center-West of Brazil

Now, another model for a Brazilian state is presented, bringing further results in the perspective of tax calibration. This DSGE model created for a Center-West state in Brazil, named Goiás, was developed by Junior, Teixeira and Silva (2022). The main objective of this author is, after building the model for Goiás (G) and the rest of the Brazilians states (REB), to analyze fiscal shocks that change one specific state tax in Brazil, named ICMS (tax on circulation of goods and services). Therefore, this regional model is an open economy one and focused on fiscal policy simulations.

Junior, Teixeira and Silva (2022) begins his model identifying each block: households, firms of intermediate goods, firms of final goods and the government, represented by a monetary authority and one state (local) fiscal policy and a federal fiscal policy. The fiscal block is very detailed, showing the local government budget costs divided by items: private goods and services bought by the local government and made available to the population, goods and services produced by the government itself, inputs for the production of public services and public investments. Revenues come from the ICMS and loans from the federal (central) government. After presenting the equations, parametrization is done via Bayesian estimation and calibration.

2.3.7.1 Results

The main result of Junior, Teixeira and Silva (2022) corresponds to what happens when a decrease in the state tax is implemented. If only the State of Goiás decreases the consumption tax (ICMS), GDP remains stable. However, when the other states do so, Goiás' output falls. Similarly, Goiás implements this fiscal policy alongside the rest of the Brazilian states, it also experiences an output decline.

This type of result is important as it shows how a model can highlight differences among regions when they implement the same fiscal policy. The national results strongly differ from the regional results, and, in this case, one specific region faces a recession while the others do not.

2.3.8 The regionalization of the SAMBA model

The SAMBA model (CASTRO et al., 2015) applied regionally is named SAMBA + REG ("REG" stands for "Regional"). This model, developed by Penna et al. (2023), analyzes a specific region of Brazil, contrasting with the national focus of the original model. Alongside Paiva (2019), Penna et al. (2023) found how aggregate shocks affect the State of Ceará. It was shown that the results are in line with previous studies, and it can be easily applied to other regions. For instance, one result showed that Ceará's regional variables are highly correlated with national variables, and the shocks differ only in magnitude.

About the model properties, it follows basically Tamegawa's way (TAMEGAWA, 2013) to disaggregate consumption into regional and foreign (world and rest of Brazil), investments and other variables, such as labor. As it is an open economy model, it is possible to observe both national and state GDP. The Taylor Rule is not only affected by regional outcomes, as there are also national variables in the model, differently from Okano et al. (2015) model for example.

2.3.9 A good idea to quantify environmental shocks in regional economies

The paper of Lim (2017) summarizes key factors to build a DSGE with the conditions to run natural resources shocks, such as a drought. This event usually occurs in agribusiness-intensive economies, such as the Brazilian State of Rio Grande do Sul, for example, ranging from every year to approximately every four to five years. For Lim (2017), the goal was to analyze if the Central Bank of the Philippines should aim to control the inflation inside the desired target or to deal with some “Dutch Disease” effects (LIM, 2017, page 2) which could happen if another country gives foreign aid to recover the Philippine economy from some natural disaster.

Differently from Lim (2017), other models could simulate natural disaster scenarios of natural disasters like floods or droughts, which results in an intense decrease in economic activity and, hence, affect the region’s public accounts, such as the State of Rio Grande do Sul government. The agriculture in Rio Grande do Sul represents around 8% of the regional GDP (IBGE, 2019) and its collected taxes are important for the maintenance of the public services and deficit control. Lim (2017) studies how the Philippine Central Bank deals with a natural disaster situation. However, if the monetary policy is centralized under a national Central Bank, the regional economy cannot rely on monetary incentives. Therefore, the economic policy would be fiscal driven to keep the economic activity stable or to maintain other objectives. According to the author, his DSGE model follows closely Peiris and Saxegaard (2007) with the new feature, which is the introduction of foreign aid to this small open economy. With the specification of “learning-by-doing” in the production of intermediate goods, it is possible to quantify how the Dutch Disease would play its role in the model. The foreign aid received is entirely spent, and the Central Bank operates in the financial market selling or buying bonds and intervening in the exchange market. In other regional models, the “foreign aid” could be simply extra central government transfers into the local one.

Lim (2017, pages 14 and 15) shows how firms’ production reacts when a natural disaster hits the economy. According to Lim (2017), “the Calvo parameter cannot remain

Table 1 – Summary of Regional DSGE models contributions and results

Author	DSGE Purpose	Motivation	Main details	Results	Parameters
Pytlarczyk (2005)	Two-Regions DSGE model for Europe and Germany.	No model for smaller regions inside EU (only for the entire Europe).	Data used from both regions for parameter calibration and to compute shocks coming from both economies.	Germany and EU behaved similar to the shocks, varying the magnitude.	Estimated.
Papageorgiou and Kazanas (2013)	Creation of a DSGE model for Greece.	Interaction of Greece with the EU.	EU is treated as the external economy.	Reductions in government expenditure improves fiscal and external imbalances.	Calibrated.
Flores et al. (2016)	Multi-Country DSGE model.	More than two-regions model to improve the analysis.	Many countries interacting at global and regional levels.	Well specified equations showed the behavior of many regions interaction.	Estimated.
Tamegawa (2013)	How to construct a regional DSGE model for any type of region.	The need of models for prefectures in Japan, states in the US or cites for example.	Local and central Governments are present distinctly in the economy and interact to make economic policies.	Shocks have different results depending on which level they come from.	Calibrated.
Tamegawa (2012)	Regional model for multi-region.	Differences in fiscal policy implementation by the central and local governments.	Households are more heterogeneous (Ricardians and Non-Ricardians) and there is a banking sector.	Fiscal shocks depend on which government implements them and where the policy is implemented.	Calibrated.
Okano et al. (2015)	Regional DSGE model for a specific region in Japan (Kansai).	No model for this specific region yet and the need to analyze its stagnation.	Focus on durable and non-durable consumption and investment specifications.	Private residential and equipment investments and productivity persistence cause Kansai stagnation.	Calibrated.
Waluyo (2017)	Two-Regions DSGE for Indonesia.	Difference in monetary policy outcomes for different regions within Indonesia.	closed economy with a system of continuum regions represented by an interval [0,1].	Central Bank of Indonesia does not react optimally to the regional shocks and desegregation enhances the analysis.	Estimated.
Paiva (2019)	Regional DSGE model for a northeast state in Brazil.	The need to ana-lyse public investment (temporary or permanents) impacts on the local economy.	closed economy with fiscal transfers from national to local government.	Sustainable and effective public investments can lead to greater potential long-run GDP.	Calibrated.
Junior, Teixeira and Silva (2022)	Regional DSGE for a center-west state in Brazil.	No DSGE for this area still and the need to quantify fiscal shocks.	Specific tax shock for local and the rest of regions economies.	If the tax shock happens only in the local economy, results change substantially.	Estimated.
Dubrovskaya, Shults and Kozonogova (2022)	Regional DSGE for Russia.	Country with continental proportions divided into 78 regions.	Focus on regional fiscal policies that may increase human capital performance.	Short-run and long-run goals are better achieved by different fiscal policies.	Calibrated.
Lim (2017)	Creation of a small DSGE model for Philippines.	How external aid might help the country when facing natural disasters.	Presence of costly adjustment costs for a specific set of firms is presented.	Costly adjustment of costs changes the dynamics of an economy when there is some natural disaster shock.	Calibrated.
Penna et al. (2023)	Expansion of the Brazilian SAMBA model to incorporate regional features inside Brazil.	The SAMBA model was used to analyse nation-wide policies only. Now, it can capture cross and within regions outcomes.	International, national and regional data used for parameter estimation. SAMBA previous framework plus regional features of the State of Ceará.	Region's economy is highly sensible to central government policies and to the national economy.	Estimated.

Source: Author's own research

constant", which is the probability of the firms adjusting their prices. This assumption implies that price adjustment is costly for the firms that produce intermediate non-traded goods, and this fact changes the economic dynamics when there is a natural disaster shock and the government spends the foreign financial aid received after the event.

2.4 IMPORTANT NOTES ON REGIONAL DSGE MODELS

This section provides important information about calibrating/estimating regional DSGE models, how to choose the most proper data and model properties. For open economy models, it is important for the researcher to clearly notice the difference between national and regional output, or foreign and regional output. As Penna et al. (2023) and Tamegawa (2013) show, there will be two (or more) outputs, consumptions, investments, and so on. Also, this point is important for the calibration or estimation of parameters, since the amount of consumption in GDP must be the proportion in relation to the desired region. In the beginning, it can be tricky because there are two output time series, two government expenditure time series, and so on, for a two-region model. This also applies for a closed economy in the government block, because there will be also two time series for government expenditure. So, clearly visualizing the difference between regions or governments is crucial to avoid mistakes.

Secondly, about the Taylor Rule for monetary policy, if a model is a closed economy, it will not have variables at national level. Hence, the Taylor Rule will be affected only by regional output and inflation. In the case that regional output and inflation is not correlated to national output and inflation, the Taylor Rule will not capture well the monetary policy dynamics and the results may alter. Also, if the sensitivity of interest to inflation is less than one, Taylor's principle will not be satisfied. It is important to watch out for the sensitivity of interest to the output gap, too: it should not be far from national Taylor Rule estimates.

Third, for the log-linearization of regional DSGE models (although it is the same procedure as standard DSGE models), the steady states can be a little different. For example, when log-linearizing Okano's model (2.3.3), the budget constraint of central and local governments have different results for the steady state of central government transfers to the local government, if government spending has different weights in output. If $\bar{G}^L$ and $\bar{G}^C$ have different values, the steady state of $\frac{Tra\bar{n}sfers}{\bar{G}^L}$ or $\frac{Tra\bar{n}sfers}{\bar{G}^C}$ will present different values[13]. A best practice is to open up every possible parameter, in opposition to the practice of "summing" them. When calibrations or priors for the parameters change, many steady states will change as well. Therefore, it is a good practice to set every parameter separately and not summing them up in one steady state. An example illustrates this situation for equation 10: The steady state of $(1-\theta_C)(\tau^C P^C C)$ can be coded this way, or set as one single parameter ($taxes_c$) which would describe how much of consumption taxes finance government spending[14]. For experience, the latter option was not the best ($(1-\theta_C)(\tau^C P^C C)$ is better than $taxes_c$).

Finally, small regions (geographically and economically) might have less data than national level data. One possibility is to use national parameters or national data, such as a proxy for the regional model. For example, if the regional data does not afford GDP in the expenditure way ($Y = C + I + G$), but there is data for G and Y, when using national data as a proxy for I, one can easily obtain C.

One last remark is that the procedure to detrend series, calibrate, and estimate parameters follows the standard DSGE models practices, which is good news. The big deal is to visualize the "extra" variables and how they interact in the model.

2.5 CONCLUSION

The current study surveys aspects of DSGE models facing the particular characteristic of describing a region. In some cases, the region is affected by the bigger economy, usually the country or continent which it is inserted, if the model is an open economy. Also, there will be two or more types of government fiscal policies will be present. In others, although the model is built as a closed economy, there are still two or more types of fiscal policies, specifically the central government (federal) and the local government (regional). For models with interaction with the foreign world, because the region is significantly smaller geographically and/or economically, there are some differences compared

[13] Note here that this type of transfers is from one government (central) to another (local), and not in relation to government transfers to households.

[14] When log-linearizing the model, $\frac{(1-\theta_C)(\tau^C P^C C)}{\bar{G}^L} = taxes_c$.

to country and continent models (such as the ones for EU) appear. The region does not have much influence over the rest of the country and many parameters differ, resulting in different simulation outcomes. Also, even if the model is built as a closed economy, the smaller region has no monetary authority, hence local economic policy is not concerned with money supply and interest rates settings. The greater possibility here is using fiscal policy to produce the desired outcomes by public planners, increasing or decreasing local expenditure or increasing or decreasing taxes, along with some public policies that may drive regional productivity to a higher level, developing its economy.

The regional DSGE models presented show that there is a vast literature concerned with regions within a country or continent, and how the local government shall behave to optimize its fiscal policy. The need to analyze specific units within a country or continent, as opposed to the national/continent level, and the fact that the results should vary significantly when narrowing the analysis, have led researchers to create these regional models. While Tamegawa (2013) created a model that shall be the starting point for any two-region model (one small region and another bigger) in the world, other authors focused on creating models for a specific and real (not theoretical) region, such as Okano et al. (2015), Japan, Pytlarczyk (2005), German, Waluyo (2017), Indonesia, Paiva (2019), northeast of Brazil, Junior, Teixeira and Silva (2022), center-west of Brazil, Dubrovskaya, Shults and Kozonogova (2022), Russia, among others studies[15].

In this perspective, regional DSGE models became notorious in the last years and are growing in number and relevance. Advancing in this topic of macroeconomic research might benefit both academia and society, when public planners enhance economic policy focused on the regional level in opposition to national objectives only, widening and deepening the horizon of analysis. The big trend now is the appearance of more and more regional DSGE models, showing new and interesting results from different macroeconomic policies.

[15] For all regional DSGE models that this research found available, the references shall give the reader sufficient information, as well as Table 1.

3 KANSAI-RS: A REGIONAL DSGE MODEL APPLIED FOR THE STATE OF RIO GRANDE DO SUL

The following essay is an application of a specific regional DSGE model (originally applied for a region in Japan) for the State of Rio Grande do Sul (RS), Brazil. After explaining the motivation to use a DSGE model for this Brazilian state, the presented model, parameter settings, results of the model moments (to verify if it is reasonable for the studied region), and possible policy outcomes show that it has interesting properties for RS and how it can be improved/extended for better results.

3.1 INTRODUCTION

This essay about the State of Rio Grande do Sul (RS) aims to use a regional DSGE model, based on Okano et al. (2015), for this specific region of Brazil (the further south state of the country, RS). This model contains some interesting differences compared to most of DSGE models built for entire countries. Firstly, this DSGE model for RS has no regional monetary authority, since the monetary authority of the country is the Central Bank of Brazil, located in Brasília, Federal District. Another fact is that this model analyzes a small economy, even smaller than Brazil (which already is a small economy compared to the USA and China, for example), with a large percentage of debt/GDP ratio that drains a significant part of its tax revenue. Hence, the results shall follow other models for countries without monetary authority and with a high debt-to-GDP ratio (for example, the model of Papageorgiou and Kazanas (2013) applied for Greece)[1]. Also, government fiscal policy is divided into local and central entities.

The current regional DSGE model could help analyze how the variables of the RS economy are related and how the model could be useful for setting future economic policies. The unique characteristics of RS, what should be the main economic policies to be implemented by the regional government and what kind of economic reforms should happen will be findings and lessons from the model. In a similar methodology using

[1] RS is currently under a fiscal regime to recover its public accounts. Its debt is almost 200% of its net tax revenue, according to (SEFAZ/RS, 2022c).

General Equilibrium as well, but the Computable one (CGE), works such as those by Porsse, Palermo and Portugal (2011), Fochezatto (2003) and Braatz (2018), studied the RS economy and showed how exogenous shocks affected the region. More specifically, for example, Porsse, Palermo and Portugal (2011) used a CGE model to simulate changes in RS tax rates and their effects on output, noticing that these shocks can change output growth positive or negatively, but with tax revenues losses in the medium and long runs.

For the present study, the research problem is divided into three key questions:

1. Why use Kansai-RS DSGE Model for the Rio Grande do Sul State?
2. Does the Kansai-RS model accurately describe the RS economy?
3. Based on the results, what should be improved for future research?

Therefore, the model can serve as a starting point to answer these questions using the DSGE framework, as well as providing a new contribution to studying the RS economy and its possible outcomes compared to other researches.

In Section 2 (Essay 1), after exposing the relevance and potential of DSGE regionalization as referenced by authors worldwide, the first natural question arises: Why use a regional model specifically for Rio Grande do Sul (RS), when there are twenty-six other units of the Brazilian Federation[2] Firstly, it shall be highlighted that RS is one of the three most indebted states in Brazil (IFI, 2020). This state has more than 200% of debt compared to its net revenue, indicating that it will take long and arduous effort to overcome this problem (the last signed recovery regime projects decades starting from 2023). Secondly, for the years between 1971 and 2020, RS local government had primary surplus only seven times. If someone, a company or a governmental entity spends more than it earns, debt accumulates. Also, If debt installments are not paid, debt accumulates even faster. This is exactly the situation of RS, which has accumulated debt owed to the

[2] Brazil is a Federation of 26 states and a Federal District. Most central government decisions come from the Federal District, such as the monetary policy setting. Rio Grande do Sul is one of the twenty-six states within the country, located in the southernmost part of Brazil, near Uruguay and Argentina.

central (or federal) government of Brazil and did not pay all its installments, suspended since 2017 by a Supreme Court decision[3].

Lastly, in 2022, by law, RS signed a regime for fiscal recovery, named Fiscal Recovery Regime (FRR). This means RS must begin to pay its debt installments to the federal government gradually from 2023 on. Hence, it is natural to have the will to study the macroeconomic dynamics of this transition, and the division between local and central government arises as a necessity for an eventual model. Some researchers may desire to study the state with the most payable debt. However, it is reasonable to focus on states in a more sensitive condition. Also, Okano et al. (2015) highlights the possibility of studying any region wanted, stating that "it is possible to compare one region's DSGE to another's. For example, separate DSGE models with the same theoretical structure can be created for Kanto and Kansai". Therefore, even if the focus for now is one (if not the most) fragile state of Brazil, other Brazilian regions can be studied further[4].

After this introduction, which presented specific aspects of the Rio Grande do Sul economy (such as its highly indebted situation) and why to use Kansai-RS model to analyze this specific region, Section 3.2 will present the Kansai-RS DSGE model, based on Okano et al. (2015). Additionally, a section will explain all the parameters used and how they were set. In Section 3.4, there will be an analysis of how the model series fit the real data, in order to verify if it has a good power to explain the real world macroeconomic dynamics of RS and some possible bias in its results. The conclusion summarizes the main results and proposes directions for future research.

---

3 In the case of Brazilian states, most of their debt is owed to the central (federal) government. Also, some of them assigned a fiscal recovery regime in 2022, instituting a formal promise to pay the full debt installments value of the debt owed to the federal government until the beginning of 2030 decade. Before, in 2017, the Supreme Court of Brazil, the STF, decided no installments should be paid until the details and technical doubts about those accumulated regional debts were completely understood and defined.

4 According to IFI (2020), RS has some of the worst fiscal indicators. RS is the state with most expenses set for public employees pensions. Also, according to the IBGE, RS population is aging in the fastest pace when compared to other Brazilian states, along with the fact that its population started to decrease in many key cities, including its capital, Porto Alegre, according to the latest IBGE census. Public health care, financial assistance, and pensions tend to increase in the following years, and the current situation is already critical. Moreover, the RS economy is strongly dependent on the agricultural sector. Frequently, droughts or heavy rains push the RS economy into recession, making it more volatile than other regional economies in Brazil. It is also important to highlight that RS public sector has around 1.5 inactive workers per active worker.

3.2 KANSAI-RS DSGE MODEL

In this section, we shall analyze the Kansai-RS DSGE model, based on Okano et al. (2015) regional DSGE model for the region of Kansai, in Japan. The Kansai-RS DSGE model, applied to the State of Rio Grande do Sul (RS), has different parameter calibrations, in order to model cycles fit the RS macroeconomic conditions, and some adaptations, especially for local and central governments equations. The Kansai-RS model has the following blocks: households, durable and non-durable goods producers, local (state and municipalities) and central (federal) governments, which run their own fiscal policies, as well as a monetary authority, which runs its monetary policy. We now turn to the Kansai-RS DSGE model equations.

3.2.1 Households

In Kansai-RS model, households derive positive utility from consuming durable and non-durable goods and negative utility from supplying labor.

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\gamma \log C_t + (1-\gamma) \log D_t - \frac{N_t^{1+\eta}}{1+\eta} \right]. \tag{12}$$

Here, C_t denotes the consumption of non-durable goods, while D_t denotes consumption of durable goods, which are land and fixed property (civil construction), according to Okano et al. (2015). The parameter γ represents the relative weight of consumption between durable and non-durable, and N_t is the labor supply with its elasticity denoted by η.

For the labor supplied by households, it is assumed imperfect substitution, following Iacoviello and Neri (2010).

$$N_t = [b^{-l}(N_t^C)^{1+l} + (1-b)^{-l}(N_t^D)^{1+l}]^{\frac{1}{1+l}}. \tag{13}$$

In this equation, N_t^C represents the quantity of labor supplied to produce non-durable goods, while N_t^D represents the labor supplied to produce durable goods. b is the relative weight that each firm uses to produce the mentioned goods. l represents the substitutability between each type of labor. According to Okano et al. (2015, page 3), "imperfect substitution of labor between producers of durable and non-durable creates wage inequality between the two sectors". Moving forward, we have the household budget constraint, which is the following:

$$\begin{aligned}(1+\tau_t^C)P_t^C C_t + P_t^D(1+\tau_t^D)I_t^D + B_t \leq \\ R_{t-1}B_{t-1} + (1-\tau_t^{WC})W_t^C N_t^C + (1-\tau_t^{WD})W_t^D N_t^D + R_t^K K_{t-1} + \\ \phi_t^C + \phi_t^D. \quad (14)\end{aligned}$$

Here, P_t^C denotes the non-durable prices, and P_t^D denotes the durable prices and I_t^D represents the amount of household investment on durable goods. W_t^C and W_t^D are the wages for producing non-durable goods and durable goods, respectively. ϕ_t^C and ϕ_t^D are the profits rate of non-durable and durable goods as well. B_{t-1} represents the bonds issued by the central government and R_{t-1} represents the nominal interest rates. $\bar{C}$, the steady state of consumption, encompasses the amount of money transferred from the governments $(\bar{Transfers}^L + \bar{Transfers}^C)$ to households, such as pensions, wages for retired employees, and financial aid, in the steady state[5]. Note here that profits, financial assets returns, wages for laboring, and the rent of capital stock must be equal or smaller than the consumption of durable and non-durable goods.

Also, the τ parameter represents taxation on durable and non-durable goods, as well as on labor income. For durable and non-durable goods, we use the D and C superscripts, and for wage for producing durable and non-durable, we have WD and WC superscripts.

Going further, there are two laws of motions: one for durable goods and another for inventory investment, which are as follows:

$$D_t = (1-\delta^D)D_{t-1} + \left[1 - S^D\left(\frac{I_t^D}{I_{t-1}^D}\right)\right] I_t^D. \quad (15)$$

$$\overline{S^D} = \overline{S^{D\prime}} = 0, \quad \overline{S^{D\prime\prime}} > 0. \quad (16)$$

[5] The reason for including a part of government spending in form of transfers to households is due to the fact that around 40-50% of government expenses are for pensions, financial aid and retired workers' wages, which are part of consumption, and the consumption share of GDP ($\frac{C}{Y}$) was low. Therefore, consumption tax contribution for government budget constraint was underestimated. When a part of government spending is transferred to households in the steady state, C increases, as well as the revenues from its taxation. As a result, taxation over labor and firms presented better tax rates.

$$K_t = (1-\delta^K)K_{t-1} + \left[1 + S^K\left(\frac{I_t^C}{I_{t-1}^C}\right)\right] I_t^C. \tag{17}$$

$$\overline{S^K} = \overline{S^{K\prime}} = 0, \quad \overline{S^{K\prime\prime}} > 0. \tag{18}$$

Here, the superscripts C denotes non-durable and D denotes durable again. D_t is the left-hand side of the law of motion for durable goods investment, while K_t represents the left-hand side law of motion for inventory investment. S is an adjustment cost and δ is the depreciation of stock of durables or inventory.

Now, we shall present the households' block Lagrangian:

$$\begin{aligned} L \equiv E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \Bigg(& \gamma \log C_t + (1-\gamma)\log D_t - \frac{N_t^{1+\eta}}{1+\eta} \\ & + \lambda_t \Bigg[R_{t-1}B_{t-1} + \left(1-\tau^{WC}\right)\frac{W_t^C}{P_t}N_t^C + \left(1-\tau^{WD}\right)\frac{W_t^D}{P_t}N_t^D + R_t^K K_{t-1} \\ & + \Phi_t^C + \Phi_t^D - \left(1+\tau^C\right)P_t^C C_t - P_t^D\left(1+\tau_t^D\right)I_t^D - B_t \Bigg] \\ & + \mu_t \left[\left(1-\delta^D\right)D_{t-1} + \left[1 - S^D\left(\frac{I_t^D}{I_{t-1}^D}\right)\right] I_t^D - D_t \right] \\ & + \lambda_t Q_t \left[\left(1-\delta^K\right)K_{t-1} + \left[1 - S^K\left(\frac{I_t^C}{I_{t-1}^C}\right)\right] I_t^C - K_t \right] \Bigg\}. \end{aligned} \tag{19}$$

The first-order conditions are the following:

$$C_t : \gamma\frac{1}{C_t} - P_t^C(1+\tau_t^C)\lambda_t = 0. \tag{20}$$

$$B_t : \lambda_t = \beta E_t R_t \lambda_{t+1}. \tag{21}$$

$$D_t : (1-\gamma)\frac{1}{D_t} - \mu_t + \beta(1-\delta^D)E_t\mu_{t+1} = 0. \tag{22}$$

$$N_t^C : (1-b)^{-l}N_t^{\eta-l}(N_t^C)^l - (1-\tau_t^{WC})\frac{W_t^C}{P_t}\lambda_t = 0. \tag{23}$$

$$N_t^D : b^{-l}N_t^{\eta-l}(N_t^D)^l - (1-\tau_t^{WD})\frac{W_t^D}{P_t}\lambda_t = 0. \tag{24}$$

$$\begin{aligned} I_t^D : -P_t^D(1-\tau_t^D)\lambda_t + \mu_t \left[1 - S^D\left(\frac{I_t^D}{I_{t-1}^D}\right) - S^{D\prime}\left(\frac{I_t^D}{I_{t-1}^D}\right)\left(\frac{I_t^D}{I_{t-1}^D}\right)\right] \\ + \beta E_t \mu_{t+1} S^{D\prime}\left(\frac{I_{t+1}^D}{I_t^D}\right)\left(\frac{I_{t+1}^D}{I_t^D}\right)^2 = 0. \end{aligned} \tag{25}$$

$$I_t^C: 1 - Q_t\left[1 - S^K\left(\frac{I_t^C}{I_{t-1}^C}\right) - S^{K'}\left(\frac{I_t^C}{I_{t-1}^C}\right)\left(\frac{I_t^C}{I_{t-1}^C}\right)\right] + \beta E_t \frac{\lambda_{t+1}}{\lambda_t} Q_{t+1} S^{K'}\left(\frac{I_{t+1}^C}{I_t^C}\right)\left(\frac{I_{t+1}^C}{I_t^C}\right)^2 = 0. \quad (26)$$

$$K_t: Q_t = \beta E_t \frac{\lambda_{t+1}}{\lambda_t}[Y_{t+1} + Q_{t+1}(1 - \delta^K)]. \quad (27)$$

3.2.2 Non-durable goods producers

Then, there is the non-durable goods producers output function, which is as follows:

$$Y_t^C(i) = A_t^C K_{t-1}^{\alpha}(i) N_t^{C(1-\alpha-\alpha_g)}(i) Kg_{t-1}^{\alpha_g} \forall i \in [0,1]. \quad (28)$$

$\alpha, \alpha_g \in [0,1]$, which means there are constant return to scale. Where Y is the output of non-durable goods, A is the level of technology used, K is the stock of capital, N is the labor force, Kg is the public stock of capital. Then, we have the cost-minimization problem of the firms:

$$\mathcal{L} = \frac{W_t^C}{P_t^C} N_t^C + \frac{R_t^K}{P_t^C} K_{t-1} - \varphi_t^C (A_t^C K_{t-1}^{\alpha}(i) N_t^{C(1-\alpha-\alpha_g)}(i) Kg_{t-1}^{\alpha_g}(i) - Y_t^C). \quad (29)$$

Here, we have a new variable, which is R_t^K, denoting the nominal rate of rental price of capital stock. φ_t^C is the real marginal cost that the firms of non-durable goods face for their production.

The first-order conditions for the non-durable goods producers' block are the following:

$$\frac{W_t^C}{P_t^C} = (1-\alpha)\frac{\varphi_t^C Y_t}{N_t^C}, \quad (30)$$

$$\frac{R_t^C}{P_t^C} = \alpha \frac{\varphi_t^C Y_t}{K_{t-1}}. \quad (31)$$

In this way,

$$\varphi_t^C = \frac{w_t^{C^{1-\alpha}} (r_t^K)^{\alpha}}{A^C Kg^{\alpha_g}}, \quad (32)$$

$$\frac{K_{t-1}}{N_t^C} = \frac{\alpha}{1-\alpha}\frac{w_t^C}{r_t^K}. \quad (33)$$

w_t^C and r_t^K with lowercase letter means they are in real terms, i.e., they are already divided by prices P_t.

3.2.3 Durable goods producers

Now, we have the following output function for the durable goods production:

$$Y_t^D(i) = A_t^D N_t^D(i) \forall i \in [0,1]. \tag{34}$$

And those firms have the following cost-minimization problem:

$$\mathcal{L} = \frac{W_t^D}{P_t^D} N_t^D - \varphi_t^D (A_t^D N_t^D(i) - Y_t^D). \tag{35}$$

The FOC is the following, deriving the Lagrangian in relation to N^D:

$$\frac{W_t^D}{P_t^D} = \varphi_t^D A_t^D. \tag{36}$$

The durable output equation is the same as the non-durable goods producer's output equation, with the difference that there is no stock of capital used in the production function. A_t^D represents the technology level used to produce durable goods, while N_t^D denotes the labor force used to produce these goods as well.

The durable goods producers' cost minimization problem results in the following first-order conditions (FOC):

$$\varphi_t^D = \frac{1}{A_t^D} \frac{W_t^D}{P_t^C} q_t^{-1}, \tag{37}$$

$$\text{where} \quad q_t = \frac{P_t^D}{P_t^C}. \tag{38}$$

In these FOCs, φ_t^D represents the real marginal cost to durable goods firms, while q_t denotes the relative price of durable goods in terms of the price of non-durable goods.

3.2.4 Calvo Pricing

In the Kansai-RS model, firms face price rigidity, implying that monetary policy may affect the output level, not just prices. Some firms adjust their prices while others do not, with a probability of $(1-\omega)$ (OKANO et al., 2015, page 11). Also, some components of Amato's rule-of-thumb are present (AMATO; LAUBACH, 2003):

$$\max \quad E_t \sum_{j=0}^{\infty} (\omega\beta)^j \left(\frac{\lambda_{t+j}}{\lambda_t}\right)^{-\sigma} \left(1-\tau^F_{t+j}\right) \Phi^z_{t+j},$$

$$\Phi^z_{t+j} \equiv \left[\left(\frac{P^z_t(i)}{P^z_{t+j}}\right) - \varphi^z_{t+j}\right] Y^z_{t+j}(i), \tag{39}$$

$$\text{subject to } Y^z_t(i) = \left(\frac{P^z_t(i)}{P^z_t}\right)^{-\theta} Y^z_{t+j}(i) \quad , \text{ for } z = C, D.$$

$$P^{z,r}_t = P^{z,*}_{t-1}\left(\frac{P^z_{t-1}}{P_{t-2}}\right), \tag{40}$$

for $z = C, D$.

With the assumptions presented above, the log-linearized New-Keynesian Philips Curve (NKPC) is as follows:

$$\pi^z_t = \gamma^z_F E_t \pi^z_{t+1} + \gamma^z_b \pi^z_{t-1} + K^z_1 \varphi^z_t + K^z_2 \tau^z_t \qquad \text{for } z = C, D. \tag{41}$$

Also,

$$\gamma^z_F \equiv \frac{\omega^K \beta}{\omega^K + \lambda^K\left(1-\omega^K(1-\beta)\right)}, \tag{42}$$

$$\gamma^z_b \equiv \frac{\lambda^K}{\omega^K + \lambda^K\left(1-\omega^K(1-\beta)\right)}, \tag{43}$$

$$K^z_1 \equiv \frac{\left(1-\omega^K\right)\left(1-\omega^K\beta\right)\left(1-\lambda^K\right)}{\omega^K + \lambda^K\left(1-\omega^K(1-\beta)\right)}, \tag{44}$$

$$K^z_2 \equiv \frac{\left(1-\omega^K\right)\left(1-\omega^K\beta\right)\left(1-\lambda^K\right)}{\omega^K + \lambda^K\left(1-\omega^K(1-\beta)\right)} \frac{\tau^F}{1-\tau^F}. \tag{45}$$

3.2.5 Monetary Policy

Also, the central bank has the following monetary policy rule, which is already log-linearized:

$$\hat{R}_t = (1-\rho_r-\rho_r^2)(\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t) + \rho_r \hat{R}_{t-1} + \rho_r^2 \hat{R}_{t-2}. \tag{46}$$

In this equation, ρ_r represents the inertia of the interest rate, $\hat{\pi}$ denotes inflation and $\hat{Y}$ signifies the output gap. If inflation is higher or output is increased, the central bank will raise interest rates to control the inflationary pressures. Additionally, if the interest rate from the previous period was higher, the current interest rate will also be higher. Appendix B presents the results for the estimation of ρ_r, ρ_r^2, ϕ_π and ϕ_y using RS data following Carvalho, Nechio and Tristão (2021).

3.2.6 Government: Important block and adaptations

For the government block, the following equations comprise the regional/local government and the national government, respectively.

$$G_t^L + IG_t^L = (1-\theta_C)\left(\frac{\tau_t^C P_t^C C_t + \tau_t^D P_t^D I_t^D}{P_t}\right) + (1-\theta_W)(\tau_t^{WC} w_t^C N_t^C + \tau_t^{WD} w_t^D N_t^D) + (1-\theta_F)\tau^F(\phi_t^C + \phi_t^D) + \omega TR_t, \quad (47)$$

where $\bar{Transfers}^L$ to households is subtracted from $\bar{G}^L$.

The first line of the above equation represents taxes on consumption of non-durable (C) and durable goods (D), while the second represents taxes on wages from work producing non-durable and durable goods, and the last one on profits from non-durable and durable goods. The last component ωTR_t represents the amount of taxes transferred from the central to the local government.

Now, turning to the central government, the equation is exactly the same, adding the fact that the national budget can issue debt in terms of bonds (B):

$$G_t^C + IG_t^C - B_{t+1} + R_t B_t + \omega TR_t = \theta_C\left(\frac{\tau_t^C P_t^C C_t + \tau_t^D P_t^D I_t^D}{P_t}\right) + \theta_W(\tau_t^{WC} w_t^C N_t^C + \tau_t^{WD} w_t^D N_t^D) + \theta_F \tau^F(\phi_t^C + \phi_t^D) + T_t, \quad (48)$$

where $\bar{Transfers}^C$ to households is subtracted from $\bar{G}^C$. Here, it is necessary to highlight that their budgets are the same, with the one difference that the federal government issues debt, and while one receives θ_z, the other receives $(1-\theta_z)$ with the taxes collected, for $z = C, W, F$. Also, ωTR_t represents the portion transferred from the central government, while T_t "is the portion of funds transferred to the central government from local governments other than the specific local government examined in the model" (OKANO et al., 2015, page 12). Φ_t^C are the profits of non-durable firms in real terms ($\frac{P_t^C Y_t^C}{P_t} - r_t^K K_{t-1} - w_t^C N_t^C$), while Φ_t^D are the profits of durable firms ($\frac{P_t^D Y_t^D}{P_t} - w_t^D N_t^D$) also in real terms. While Okano et al. (2015) do not include the central government expenditure in output, our model incorporates this feature, such as Tamegawa (2013). Additionally, our model incorporates public investment (IG_t^C and IG_t^L) and monetary transfers, such as pensions for retired workers and financial aid ($Transfers$), in the steady state of C, G^L and G^C.

3.2.7 Equilibrium conditions

Now, there are equations describing the equilibrium of the model for output, labor, and goods markets. The equations are the following ones:

$$Y_t = Y_t^C + Y_t^D, \tag{49}$$

$$Y_t = C_t + I_t^D + I_t^C + G_t^L + G_t^C + IG_t^L + IG_t^C, \tag{50}$$

$$N_t = N_t^C + N_t^D, \tag{51}$$

$$B_t = 0, \tag{52}$$

$$N_t^K = \int_0^1 N_t^K(i)di, \tag{53}$$

for K = C, D.

$$K_t = \int_0^1 K_t(i)di. \tag{54}$$

Then, the presented optimized system of equations, from the first order conditions, is log-linearized (using an approximation of order one) around the steady state.

3.3 PARAMETRIZATION

In this section, all calibrated or estimated parameters (by econometric techniques) of Kansai-RS model are presented. These parameters are based on international and national (Brazilian authors) literature, as well as historical data collected from governmental institutions such as the "Brazilian Institute of Geography and Statistics", named IBGE, and the Department of Economics and Statistic of the RS Government, named DEE/RS. The parameters are set in annual frequency (e.g., depreciation rate) or percentage of another variable (e.g., government consumption share of output). Table 2 shows their set values and sources[6].

The discount factor (β) is calibrated to the historical average of the Brazilian basic interest rate, named Selic, divided by regional inflation calculated by IBGE from 2011 to 2019. This results in an average of 4.35% of real interest rates. According to Issler, Piqueira et al. (2001, page 26), β typically ranges from 0.88 to 0.96 in annually, with a value of approximately 0.9585 used in the model ($\beta = \frac{1}{\bar{R}}$, where $\bar{R}$ is 1.0435).Issler, Piqueira et al. (2001) states that the annual discount rates in Brazil are close to 0.90 in annual basis, whereas in the United States, they are close to 1.0: "comparing the Brazilians consumers to the American ones, it is clear that the Brazilian ones are more impatient".

For the depreciation rate parameters (δ_k and δ_g), which are the depreciation of private capital stock and depreciation of public capital stock, they are set at 5% per year, according to the Penn World Tables data average from 2011 to 2019 (FEENSTRA; INKLAAR; TIMMER, 2015)[7]. Carvalho et al. (1996, page 20) estimates an approximate annual depreciation rate of 4.32% for Brazil, aligning closely with the chosen value. Also, this yields a value of approximately 2.5 capital/output ratio, following IPEA (2024, page 3). The public capital stock is estimated at approximately 0.5 of output, following Frischtak and Mourão (2017, page 132).

For the production function, we follow Filho, Pessôa and Veloso (2010), who claims that α plus α_g is around 0.4 in the production function of the Brazilian firms, hence capital stock represents around 40%, while labor represents 60% for the output of non-durable goods, disregarding technology/efficiency (A^C) in the equation[8].

$\bar{G}^L$, $\bar{G}^C$, $\bar{IG}^L$ and $\bar{IG}^C$ represent the steady states share of the types of government

6 For alternate options for calibration, Cavalcanti and Vereda (2011) presents various references for the Brazilian economy, comparing Brazilian and international literature. There are plenty of possibilities, and Kansai-RS model will be calibrated to best describe the RS economy inside the reasonable zone for parameters values.

7 Data available at available for download at www.ggdc.net/pwt.

8 Also, regional accounts of RS shows that the historical average of workers' total earnings (wages multiplied by labor) in relation to output is 40%. Setting these parameters alongside the others (e.g., depreciation rate and durable and non-durable shares of output) resulted in the model steady state of workers' earnings compared to output to be the same as the data average value.

expenditure in GDP between the years of 2011 and 2019 in RS and in Brazil, according to the historical data series from governmental entities (FINBRA, 2024) (NTB, 2023a)[9].

The elasticities of durable and non-durable investment are set in 1.25 (ψ and ψ_C), in annual terms, according to Cavalcanti and Vereda (2011)[10]. The Inverse-Frisch elasticity of labor supply is set at 1.94, following Almeida et al. (2011).

The share of durable goods inflation in the Consumer Price Index (CPI) is set at 0.15, based on inflation data of IBGE (2023a). The share of durable in GDP follows the national accounts data with an average of DEE/RS (2022), and is set for 0.05.

For the monetary policy block, we estimate the parameters following Carvalho, Nechio and Tristão (2021), who shows it is possible to estimate monetary policy coefficients using the Ordinary Least Squares estimator (OLS). The results are approximately 0.55 for the interest autoregressive component, 1.75 for the elasticity of interest to inflation, and 0.65 for the elasticity of interest to output gap. Those values are close to national estimates, and Taylor Rule principle of $\rho_\pi > 1$ is satisfied.

The share of housing investment (durable goods) equals durable GDP. Inventory investment (non-durable goods) is set according to national investment data minus RS housing investment, as RS GDP is not calculated by the expenditure view. Consumption is set by subtracting from 1 all other components ($\bar{C} = \bar{Y} - \bar{I}^C - \bar{I}^D - \bar{G}^L - \bar{G}^C - \bar{IG}^L - \bar{IG}^C + \bar{Transfers}^L + \bar{Transfers}^C$)[11].

The taxes rate, the share between local and central governments revenues for

[9] The Local government expenditure share in RS output (around 15% of state GDP), was calculated as the average of 2011–2019 data, summing all committed expenses of the 497 cities of the state (around 6 to 7% of the state GDP) plus state government expenditure (around 7% of the state GDP). Government transfers of income to households, $\bar{Transfers}^L$ and $\bar{Transfers}^G$, are around 40% and 50% of $\bar{G}^L$ and $\bar{G}^C$, respectively.

[10] Dynare community recommends dividing by four adjustment costs to turn quarterly frequency into annual frequency.

[11] Since a significant part of the local and central governments budget is transferred to households, such as pensions for retired workers and financial aid, there was the need to include this type of revenue for households in order to increase $\bar{C}$ in the steady state share of GDP. This adjustment allows more contributions from consumption taxes for the government's budget and reduces the adjustment allows. During the simulation about government spending, however, it is considered that the shock is on the part of G_t that does not include transfers for households. A possibility for future research is to divide those variables in order to account for the cycle of transfers as well.

consumption, income and corporate taxes, and how much each type of tax (consumption, income and corporate taxes) contributes to the governmental budget are set based on the FINBRA (2024) data. According to the data, approximately 70% of local government spending comes from consumption taxes, 16% from central government transfers and the rest from income taxes (primarily from public employees and social contributions for retirement and health care). From these values, it was possible to calculate the tax rates over consumption and over labor. Using data from RFB (2024) and FINBRA (2024), it was also possible to calculate the share of consumption and labor taxes that belongs to each type of government. Local governments receive around 90% of consumption taxes, while central government receives around 85% of labor taxes. Taxes over firms are entirely for central government, hence $\theta_F = 1$. This scenario results in tax rates of: $\tau^C = \tau^D = 0.21$ and $\tau_{WC} = \tau_{WD} = 0.24$, with uniform taxation on workers regardless of the sector they work in and equalized consumption taxes on durable and non-durable goods for simplicity. It is also known from RFB (2024) that around 45% of central spending come from income taxes (wages and social contributions), 45% from firms and the rest over consumption, which is the minority of the collected taxes.

Also, some additional parameters are set in order for the model to be able to demonstrate properly the RS variables cycles. λ_c and λ_d, respectively the share of rule-of-thumb firms in non-durable and durable sectors, are set after the results showed to be appropriate. ζ comes from Castro et al. (2015), around 0.4. Calvo lottery parameters are set to around the fourth power of 0.66, turning this usual parameter value to annual frequency.

The parameter I_l represents the labor disutility of switching sectors between durable and non-durable goods, and is sensitive for the model. To represent properly the second moments of the model, it was calibrated to 0.85. This lead the model to have upward output movements and wage movements after a productivity shock for each sector.

Table 2 – Kansai-RS model parameters (Annual frequency)

	Parameter Description	Value	Source
β	Discount Factor	0.9585	Historical Average
δ_k	Depreciation rate of private capital stock	0.05	Penn World Tables
δ_g	Depreciation rate of public capital stock	0.05	Carvalho et al. (1996)
α_g	Productivity effect of social capital	0.05	Filho, Pessôa and Veloso (2010)
α	Production Function	0.35	Filho, Pessôa and Veloso (2010)
ψ	Elasticity of durable investment	1.25	Cavalcanti and Vereda (2011)
ψ_C	Elasticity of non-durable investment	1.25	Cavalcanti and Vereda (2011)
ϕ	Inverse-Frisch elasticity of labor supply	2.5	Almeida et al. (2011)
I_l	Labor disutility of switching sectors	0.85	Author's calibration.
γ	Share of durable goods inflation in CPI	0.15	IBGE (2023a)
α_Y	Share of durables in GDP	0.05	DEE/RS (2022)
θ_c	Calvo lottery for non-durable	0.05	Cavalcanti and Vereda (2011)
θ_d	Calvo lottery for durable	0.05	Cavalcanti and Vereda (2011)
λ_c	Share of rule-of-thumb firms in non-durable sector	0.85	Author's calibration
λ_d	Share of rule-of-thumb firms in durable sector	0.855	Author's calibration
ζ	Share of Non-Ricardians Households	0.4	Castro et al. (2015)
ϕ_π	Inflation response in monetary policy rule	1.75	Appendix B
ϕ_y	Output response in monetary policy rule	0.65	Appendix B
ρ_r	Interest rate smoothing $(t-1)$	0.95	Appendix B
ρ_r^2	Interest rate smoothing $(t-2)$	-0.40	Appendix B
	Steady State Shares		
$\bar{I}^D$	Share of housing investment	0.05	DEE/RS (2022)
$\bar{I}^C$	Share of investment	0.1285	IBGE (2023b)
$\bar{C}$	Share of consumption (including government transfers)	0.6	Author's calculations
$\bar{G}^L$	Share of local government expenditure	0.113	FINBRA (2024)
$\bar{G}^C$	Share of central government expenditure	0.087	NTB (2023a)
$\bar{IG}^L$	Share of local government investment	0.0137	FINBRA (2024)
$\bar{IG}^C$	Share of central government investment	0.014	NTB (2023a)
$\bar{Tr}^L$	Local transfers to households expenditure	0.04	FINBRA (2024)
$\bar{Tr}^C$	Central transfers to households expenditure	0.08	NTB (2023a)

Source: Author's own research (2024)

Table 3 – Kansai-RS model parameters — Fiscal and persistence parameters

	Parameter Description	Value	Source
	Tax parameters		
τ_C	Consumption tax of non-durable goods	0.21	Based on other parameters
τ_D	Consumption tax of durable goods	0.21	Based on other parameters
τ_{WC}	Income tax for worker in non-durable goods firms	0.24	Based on other parameters
τ_{WD}	Income tax for worker in durable goods firms	0.24	Based on other parameters
τ_I	Corporate tax	0.275	Based on other parameters
	Budget Parameters		
ϕ_C^L	Share of consumption tax on local government budget	0.72	FINBRA (2024)
ϕ_W^L	Share of income tax on local government budget	0.12	FINBRA (2024)
ϕ_C^C	Share of consumption tax on central government budget	0.08	FINBRA (2024)
ϕ_W^C	Share of income tax on central government budget	0.45	FINBRA (2024)
ϕ_{TR}	Share of transfers from central government to local government	0.025 of GDP	FINBRA (2024)
θ_C^L	Percentage of collected consumption tax for local government	0.9	FINBRA (2024)
θ_W^L	Percentage of collected income tax for local government	0.15	FINBRA (2024)
	Persistence parameters[a]		
ρ_{A_C}	Persistence of productivity shock (non-durable)	None	Estimated with data
ρ_{A_D}	Persistence of productivity shock (durable)	None	Estimated with data
ρ_{G^L}	Persistence of local government spending	0.45	Estimated with data
ρ_{G^C}	Persistence of central government spending	None	Estimated with data
ρ_{IG^L}	Persistence of local government investment	None	Estimated with data
ρ_{IG^C}	Persistence of central government investment	None	Estimated with data

Source: Author's own research (2024)

[a] Since the data is in annual frequency, most of AR(1) coefficients were not significant. Also, the series presented strong random behaviors, varying intensively from year to year.

3.4 DOES THE KANSAI-RS MODEL EXPLAIN THE RIO GRANDE DO SUL ECONOMY? COMPARISON OF THE MODEL AND DATA MOMENTS AND IRFS

In this section, we analyze the differences between the data generated by the Kansai-RS model and the real data from the State of Rio Grande do Sul. Since the model operates in terms of log-deviations from the steady state, we need to compare the model-generated data with the cycles of RS, which are measured by the difference between the historical series (log-data) against the trends of the analyzed variables. We used the linear trend (estimated by OLS) to measure these values, resulting in the cycle of each variable (Annexes A and B show all series used for econometric estimations, including productivity shocks).

To compare the model-generated data and the real data from RS, we use the second moments (standard-deviation and correlations) of both types of data. The observed series of the RS economy include: i) Output; ii) Wages; iii) Inflation; iv) Local government spending; v) Local government investment; vi) Central government spending; and, finally, vii) Central government investment. They are collected from DEE/RS (2022) for the State output, FINBRA (2024) and NTB (2023b), for local and central government spending and investment series, IBGE (2023a) for local inflation, represented by Porto Alegre inflation (capital of RS), and IBGE (2023c), for the regional total wages. Linear trend separates the trend and cycle of these series, and they were previously deflated in order to do so.

We will run two types of simulations to compare the model moments with observed data moments:

1. One shock at a time, adjusting key parameters until the simulations are appropriate (i.e., according to economic theory and data moments).
2. A sequence of several types of shocks using values from data cycles series.

To perform these comparisons, we will follow Kydland and Prescott (1982), Sbordone et al. (2010) and Leite (2019) to compare the moments of the simulations. Their works use different parameter calibrations to compare the results of the theoretical model moments with the original data moments, using standard-deviation, correlation, and sign of the results as benchmarks. Since the Kansai-RS model is a calibrated model, this will be the chosen method.

Five simulations will be done: government expenditure shock (local and central), productivity shock for non-durable and durable (housing, lands and fixed property) products, and local and central governments shocks together.

For the second type of simulation, which involves running a sequence of several types of shocks for different exogenous variables, Costa-Filho (2022) shows how this can be done in a simple way. About the estimation of the shocks, Brinca, Filho and Loria (2023) show an interesting procedure: estimating an AR(1) process from the cycle of the series, and the residuals are the respective shocks. If the AR(1) coefficient was not significant, it was considered zero, and the shocks were considered the total amount of the cycle. This happened especially for productivity shocks, where events such as droughts, national recessions and pandemics explain the randomness of those series. Also, those periods are based on events of table 4, which classifies them in terms of when they occurred, direction of the impact in output, whether they are supply, demand or both types of shocks, their magnitude and, how they are set inside the model. The estimated shocks came from the series in Annexes A and B.

Table 4 shows all macroeconomic events of Rio Grande do Sul from 2012 to 2022, starting with a huge drought that was faced in 2012 to another drought in 2022, with a short amount of time from the big recession caused by the pandemics in 2020 and 2021. Mapping these events is important to perform the second proposed exercise. The generated series of output will then be compared to the total amount of earned wages ($W = w^C N^C + w^D N^D$, which are the costs of labor for firms) and inflation in the RS economy[12].

It is necessary to highlight that the model has adaptations in the government fiscal block, GDP identity, and has public stock of capital and Ricardians households. Okano et

[12] Since w is the real wage, W is the sum of $w^C N^C + w^D N^D$ compared to deflated data.

Table 4 – Shocks to the RS economy during 2012-2022 period

Event	Year	Shock	Sign	Size	Further information
Drought	2012	Supply	Negative	Intense	It can describe in the model as a productivity shock.
Fiscal expansion	2013	Demand	Positive	Significant	It can describe in the model as a local government expenditure shock.
Agro-Boom	2013	Supply	Positive	Significant	It can describe in the model as a productivity shock.
Nation crisis	2015	Both	Negative	Significant	It can describe in the model as consumption and investment shocks to the local economy (plus bad news for expectations).
Tax hike	2016	Demand	Negative	Medium	It can be described as an upward local tax shock.
Agro-Boom	2019	Supply	Positive	Medium	It is described in the model as a productivity shock.
Pandemics and Drought	2020	Both	Negative	Intense	It can be described as productivity shocks, plus demand side decreases (consumption and investment, for example).
Agro-Boom	2021	Supply	Positive	Intense	It is described in the model as a productivity shock.
Drought	2022	Supply	Negative	Medium	It is described in the model as a productivity shock.

Source: DEE/RS (2022), FIERGS (2022) and SEFAZ/RS (2022b)

al. (2015) also suggests this type of analysis for other works[13]:

> "[. . .] it is possible to compare one region's DSGE to another's. For example, separate DSGE models with the same theoretical structure can be created for Kanto and Kansai. Regional differences would be expressed in different parameter calibrations and calibrated with reference to regional data from previous empirical research such that the models would somewhat reflect each area's respective characteristics" (OKANO et al., 2015, page 4).

3.4.1 Moments Comparison: Correlations between observed and model cycles

After the presentation of real data moments of RS, the simulated theoretical moments will be presented using fiscal and productivity shocks (all shocks are of 1 standard-deviation).

Output and local government spending are procyclical, as well as wages and inflation.

[13] Okano et al. (2015) sent their original Dynare codes, and this academic exchange was a great contribution to the understanding and adaptation of the model. The whole code provided by Okano et al. (2015) can be promptly requested to the author.

Table 5 – Correlations of RS Data

Variable	Y	G^L	IG^L	G^C	IG^C	W	π_{POA}
Y	1.00	0.38	0.1	-0.38	-0.16	0.62	0.27
G^L	0.38	1.00	-0.42	0.457	0.312	0.907	-0.085
IG^L	0.1	-0.42	1.00	-0.223	0.219	-0.475	0.256
G^C	-0.38	0.457	-0.223	1.00	0.75	0.175	-0.505
IG^C	-0.16	0.312	0.219	0.75	1.00	0.112	-0.164
W	0.62	0.907	-0.475	0.175	0.112	1.00	-0.0229
π_{POA}	0.27	-0.085	0.256	-0.505	-0.164	-0.0229	1.00

Source: Author's calculations

Table 6 – Standard-Deviation of RS Data

Variable	Y	G^L	IG^L	G^C	IG^C	W	π_{POA}
Standard-Deviation	0.032	0.00397	0.2014	0.052	0.2195	0.0555	0.024

Source: Author's calculations

On the other hand, output and central government expenditure are countercyclical. This is easily explained: when the local economy grows, local government receives more taxes and spends more. From the same perspective, when output grows, wages exhibit the same behavior. However, central government spending is countercyclical, since it may try to smooth the economy cycle (more spending in recessions, such as the pandemic, and less during normal times).

It is necessary to highlight that local inflation is negatively correlated to wages. Local government and central government have different impacts on wages as well (the magnitudes are higher for local spending, lower for central government spending and counter-cyclical with local investment). Additionally, central government is negatively correlated to inflation. It is not usual in economic theory, where increases in government spending usually raise inflation rates via demand increases.

3.4.1.1 Results of productivity shocks: Non-durable

Non-durable shock is quite similar to the durable shock. However, they differ in the magnitude of the second moment of correlation.The non-durable shock aligns closely with the real data correlation of output in relation to wages. Perhaps, this is due to the fact that

Table 7 – Correlations of Simulated Data: Non-durable productivity shock

Variable	Output	Wages	Inflation
Y	1.00	0.65	-0.71
W	0.65	1.00	0.07
π_{POA}	-0.71	0.07	1.00

Source: Author's calculations

Table 8 – Standard-Deviation of Simulated Data: Non-durable productivity shock

Variable	Output	Wages	Inflation
Standard-Deviation	0.82	0.36	1.66

Source: Author's calculations

non-durable goods are the most part of the regional economy output, thus a non-durable shock creates a closer output and wages dynamic than a durable shock. Again, wages are less volatile than expected by real data, but inflation is more volatile. In summary, productivity shocks exhibit correlations close to those in real data variables, but they are slightly more volatile than expected.

3.4.1.2 Results of productivity shocks: Durable

Table 9 – Correlations of Simulated Data: Durable productivity shock

Variable	Output	Wages	Inflation
Y	1.00	0.77	-0.72
W	0.77	1.00	-0.13
π_{POA}	-0.77	-0.13	1.00

Source: Author's calculations

Table 10 – Standard-Deviation of Simulated Data: Durable productivity shock

Variable	Output	Wages	Inflation
Standard-Deviation	0.042	0.02	0.08

Source: Author's calculations

For a durable productivity shock, positive results appear. Output is positively correlated with wages, and negatively correlated with local inflation, as theory predicts. Wages and inflation are negatively correlated. This is due to the fact that wages rose due to an upward productivity shock, which means workers are producing better, hence goods and services might be cheaper.

For the standard-deviation of simulated data, we see that wages are less volatile in comparison to output. On the other hand, inflation is more volatile than real data standard-deviation.

3.4.1.3 Results of Local Government Fiscal Shock

Local government spending shock is highly correlated to output, as indicated by real data. However, real data shows it should be negatively correlated with inflation. We suppose this discrepancy arises because the historical series are not so long. Actually, it is reasonable to suppose local government expenditure shock increases inflation by uprising demand for goods and services.

Regarding local government investment shocks, they are almost not correlated to inflation, because they increase output capacity, thereby offsetting prices increases (and expanding supply). Also, it is almost not correlated with wages, while real data present negative correlation.

For the standard-deviation, local government spending is more volatile than output, and real data shows (for that sample period) that they should be very close.

Table 11 – Correlations of Simulated Data: Local government shock

Variable	Y	W	π_{POA}	G^L	IG^L
Y	1.00	0.93	0.67	0.81	0.15
W	0.93	1.00	0.85	0.70	0.15
π_{POA}	0.67	0.85	1.00	0.64	0.07
G^L	0.81	0.70	0.64	1.00	-0.05
IG^L	0.15	0.15	0.07	-0.05	1.00

Source: Author's calculations

3.4.1.4 Results of Central Government Fiscal Shock

The central government shock is similar to the local government shock. Real data suggests it should be countercyclical, which it is reasonable according to economic theory.

Table 12 – Standard-Deviation of Simulated Data: Local government shock

Variable	Y	W	π_{POA}	G^L	IG^L
Standard-Deviation	0.06	0.04	0.10	1.21	1.08

Source: Author's calculations

However, it may not necessarily be a causality situation that the model can capture. It would not be logical for the model to indicate a decrease in output when central government increases its expenditure, at least in the first periods. Also, inflation is very procyclical in relation to both local and central governments shocks, which is quite reasonable (it would not make sense for inflation to decrease when government spending increases). For the standard-deviation second moment, the model exhibits significant volatility for the governments fiscal shocks compared to real data results.

Table 13 – Correlations of Simulated Data: Central government shock

Variable	Y	W	π_{POA}	G^C	IG^C
Y	1.00	0.94	0.89	0.90	0.16
W	0.94	1.00	0.98	0.76	0.14
π_{POA}	0.89	0.98	1.00	0.74	0.11
G^C	0.90	0.76	0.74	1.00	-0.02
IG^C	0.16	0.14	0.11	-0.02	1.00

Source: Author's calculations

Table 14 – Standard-Deviation of Simulated Data: Central government shock

Variable	Y	W	π_{POA}	G^C	IG^C
Standard-Deviation	0.05	0.04	0.12	1.08	1.08

Source: Author's calculations

3.4.1.5 Results of Both Governments Fiscal Shock

When both local and central governments increase their expenditure above steady state values, the results are similar when only one of them initiates a shock. Wages continue correlated with output in this simulation compared to local and central governments shocks. Inflation is strongly correlated to local and central governments and output.

Table 15 – Correlations of Simulated Data: Local and central governments shock

Variable	Y	W	π_{POA}	G^L	IG^L	G^C	IG^C
Y	1.00	0.93	0.82	0.69	-0.05	0.54	0.18
W	0.93	1.00	0.95	0.52	-0.08	0.54	0.20
π_{POA}	0.82	0.95	1.00	0.39	-0.07	0.58	0.21
G^L	0.69	0.52	0.39	1.00	0.06	-0.06	-0.01
IG^L	-0.05	-0.08	-0.07	0.06	1.00	-0.18	-0.04
G^C	0.54	0.54	0.58	-0.06	-0.18	1.00	0.13
IG^C	0.18	0.20	0.21	-0.01	-0.04	0.13	1.00

Source: Author's calculations

Table 16 – Standard-Deviation of Simulated Data: Local and central government spending shock

Variable	Y	W	π_{POA}	G^L	IG^L	G^C	IG^C
Standard-Deviation	0.08	0.06	0.17	1.28	0.98	1.07	0.97

Source: Author's calculations

3.4.1.6 Variance and Historical Decomposition of RS Output Cycle

The variance and historical decomposition of the RS output cycle show interesting results. As shown in Table 17 and Graph 1, most of RS's output dynamics are attributed to productivity shocks. However, interest and government (local and central) spending shocks are also relevant. It is possible to verify that in 2015 and 2020, the years when recessions hit their lowest levels, most of them were due to non-durable and durable productivity shocks.

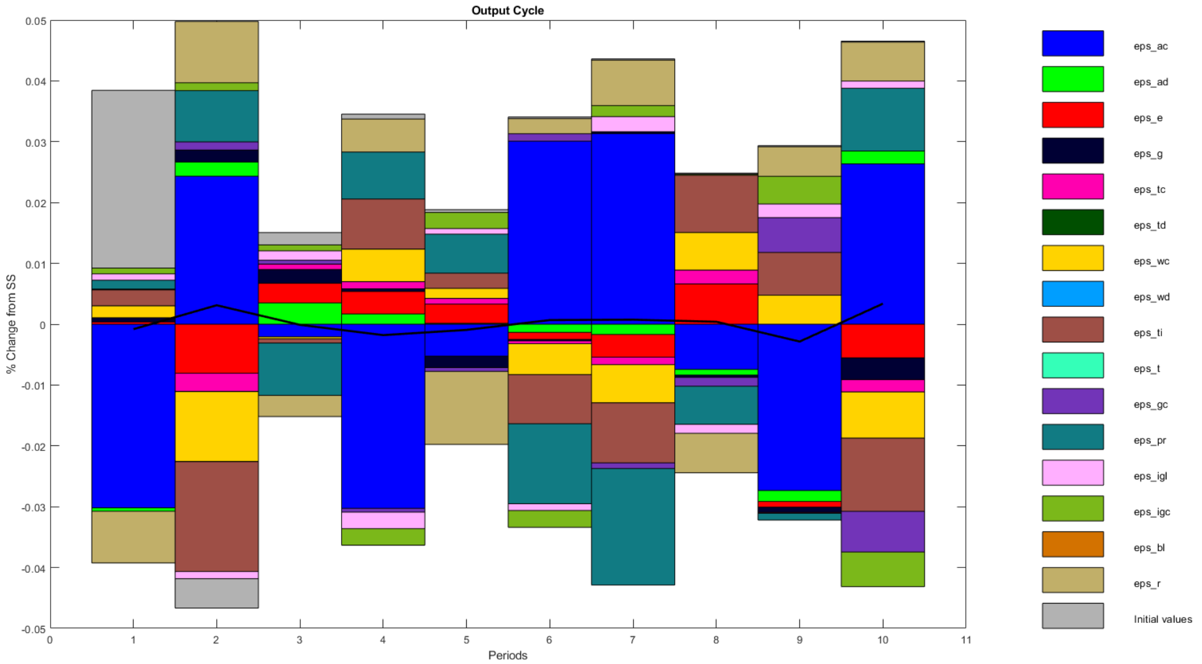
On the other hand, interest rates (Selic), local government spending, central government spending and other minor shocks, such as households preferences and taxes, helped to explain the cycle as well.

Table 17 – Variance Decomposition, simulating one shock at a time (in percent)

Shock in	A^C	A^D	G^L	IG^L	G^C	IG^C
Y	102.64	0.24	0.76	0.01	0.45	0.02
W	99.62	0.28	1.64	0.04	1.42	0.05
π_{POA}	98.15	0.24	0.54	0.02	0.63	0.02

Source: Author's calculations

Figure 1 – RS Output Cycle Decomposition — 2012-2021



Source: Author’s calculations (2024)

3.4.2 Results of Simulated Sequence of Shocks

For this type of simulation, six shocks hit the RS economy in order to simulate the real data between 2012 and 2021, derived from the residuals of the estimations from the original series shown in Annexes A and B. Table 4 also shows all macroeconomic events within the state economy. For the productivity shocks, it was made a growth decomposition between capital, labor, and productivity over output growth[14]. The shocks are the following:

1. Local government expenditure shock.

2. Central government expenditure shock.

3. Local government investment shock.

4. Central government investment shock.

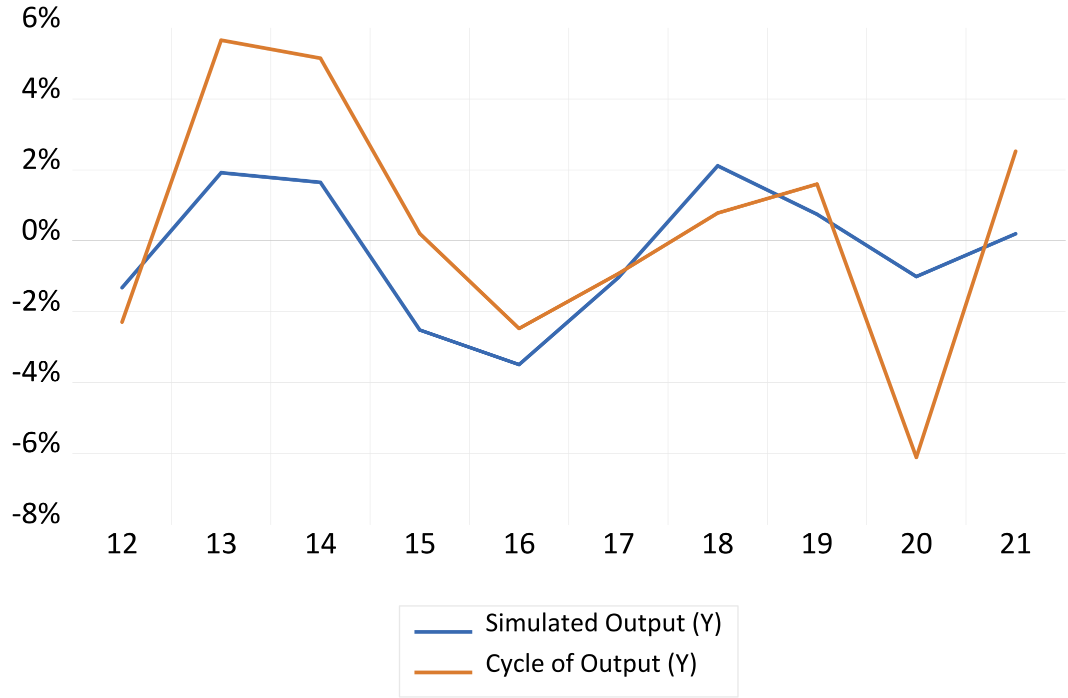
[14] For the creation of A^C and A^D series, labor was first decomposed according to IBGE (2024b) and CBIC (2024), dividing labor growth between the two types of sectors. Later, using the perpetual inventory method, it was estimated the stock of capital of non-durable sector. Subtracting labor and capital growth from output growth, the productivity series is created.

5. Non-durable productivity shock.

6. Durable productivity shock.

The estimated shocks reasonably represented the random part of the series. Additionally, the mapping of events and their intensities aligns with the estimation results of the shocks. Therefore, both the quantitative and qualitative research of the shocks are in harmony, indicating that the Kansai-RS model captured the dynamics of real data from Rio Grande do Sul.

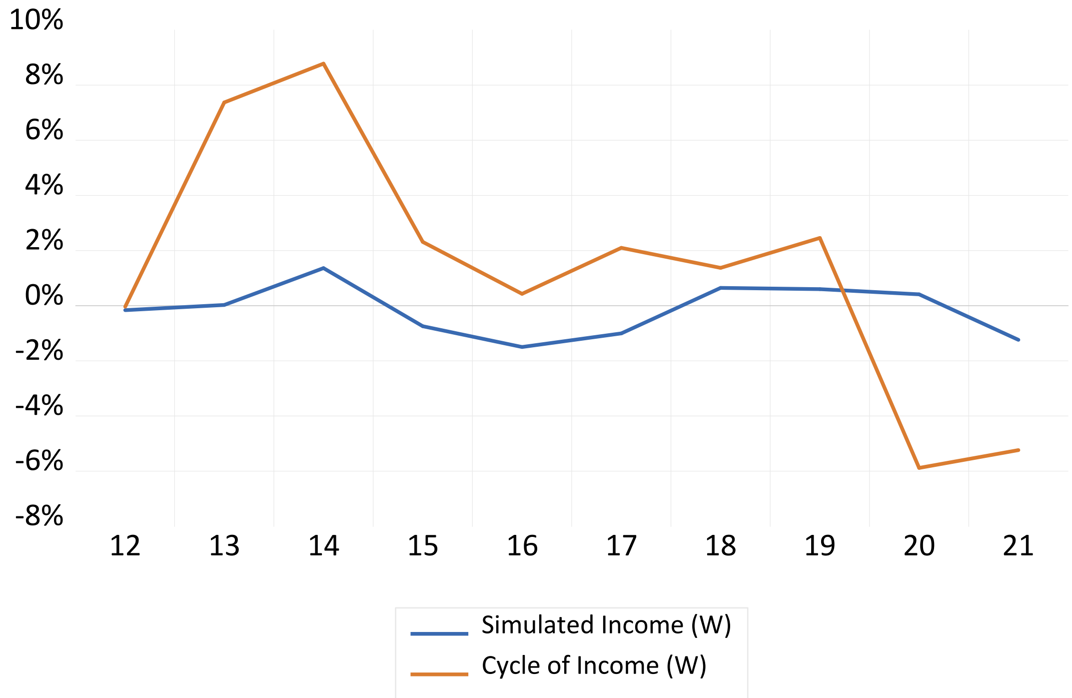
Figure 2 – RS Simulated Output and Cycle of Output - 2012-2020



Source: Author's calculations (2024)

After running all exogenous shocks, we compared the simulated sequences with the real data series, from the years of 2012 to 2021, this way: correlations between the model variables and correlations between model and actual variables. The series we used for the comparison were output, inflation and total workers' income ($W = w^C N^C + w^D N^D$), collected from IBGE (2023a) and IBGE (2023c).

Figure 3 – RS Simulated Wages and Cycle of Wages - 2012-2020

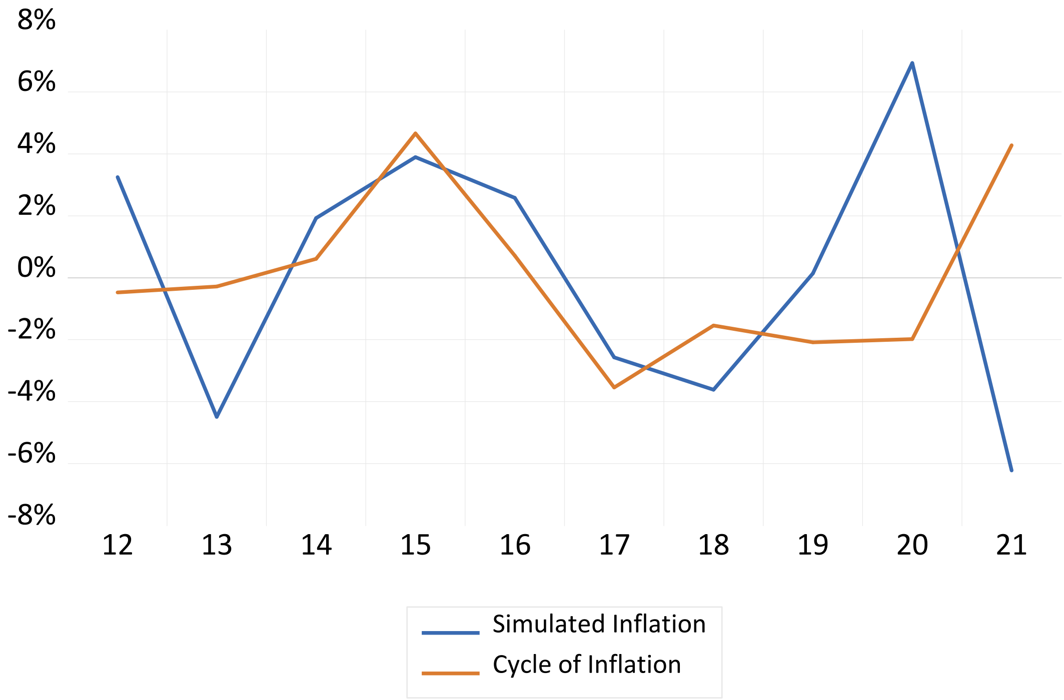


Source: Author's calculations (2024)

Comparing to the regional economy output, the movements are quietly appropriate. For inflation and wages, the results are reasonable as well. However, when the pandemic hit the economy, the model did not appropriately explain what really happened to inflation and wages. Initially, the values of the inflation series are also somewhat different from what the data shows. Besides these moments, the model captured relatively well the dynamics of the real data cycle movements.

For the moments comparison, wages are procyclical in relation to output, as they should be. Also, simulated inflation delivered a good result in terms of real data moments. In terms of standard-deviations, the model presented good results compared to real data as well. Output is very close to data's standard-deviation, as well as inflation and wages.

Figure 4 – RS Simulated Inflation and Cycle of Inflation- 2012-2020



Source: Author's calculations (2024)

Table 18 – Correlations of Simulated Sequence of Shocks

Variable	Output	W	π_{POA}
Y	1.00	0.73	-0.49
W	0.73	1.00	0.22
Inflation	-0.49	0.22	1.00

Source: Author's calculations

Table 19 – Standard-Deviation of Simulated Sequence of Shocks

Variable	Output	W	π_{POA}
Standard-Deviation	0.019	0.009	0.04

Source: Author's calculations

3.4.3 Impulse-Response Functions

The IRF for a local government shock showed an increase in output, in wages, and inflation. It is interesting that wages increase in the first period, then decrease, and then

Table 20 – Correlations of Simulated and Observed Series (Not including the Pandemics)

Variable	Correlation
Model and Actual Output	0.78
Model and Actual W	0.59
Model and Actual π_{POA}	0.57

Source: Author's calculations

return to a slightly below steady state level. For inflation, it increases and then falls below the steady state (SS) level. Later, it remains above SS.

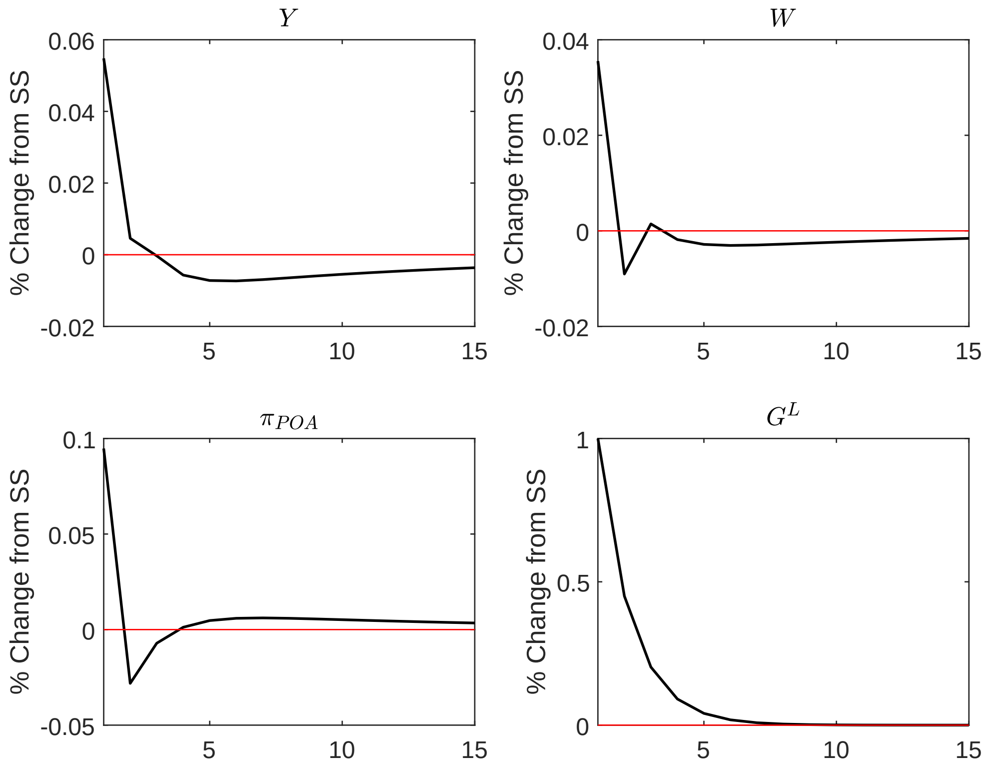
On the other hand, a local government investment shock has a different behavior, since it made inflation fall below SS after the first periods. Since IG^L increases the economy's stock of capital, the supply grows and decreases prices. However, there is still positive inflation in the first periods since demand increased.

For productivity shocks, the scenario is more optimistic if they are positive. A^C and A^D boost output and wages, and lower inflation (as expected). After two-three periods, the variables reach their steady state again. In the simulated sequence of shocks (Subsection 3.4.2), it is possible to notice the relevance of positive and negative productivity shocks conducting the RS output[15].

In summary, the simulations show that the local government can boost economic growth in the short-run, as well as increase inflation rates. Wages react similarly to output since they are very correlated with each other. Inflation reacted according to economic theory: more demand rises prices, while positive supply shocks lower inflation.

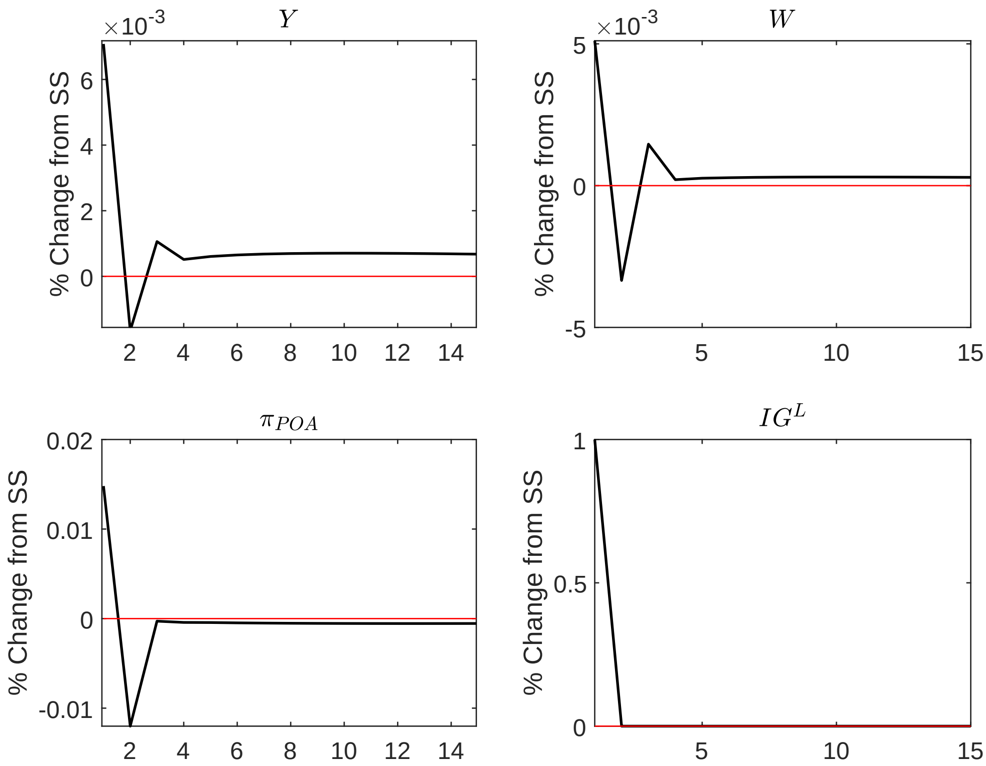
[15] The time Series of A^C and A^D are in Annex B.

Figure 5 – Shock of Local Government Spending to Output, Wages, and Inflation - G^L



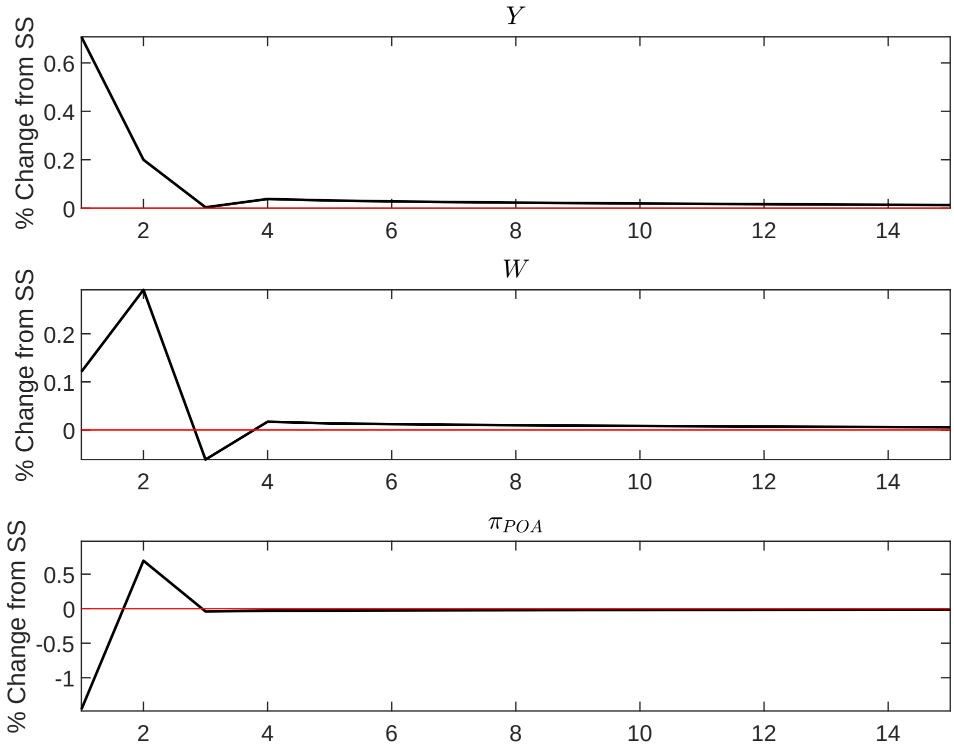
Source: Author's calculations (2024)

Figure 6 – Shock of Local Government Investment to Output, Wages, and Inflation - IG^L



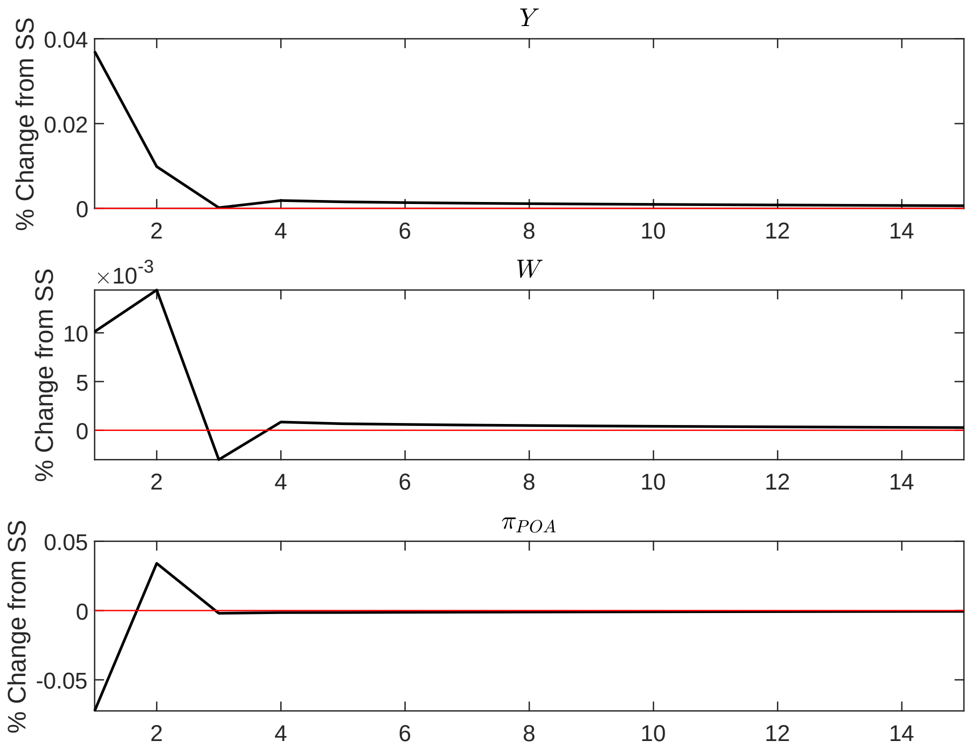
Source: Author's calculations (2024)

Figure 7 – Shock of Non-durable Sector productivity to Output, Wages, and Inflation - A^C



Source: Author's calculations (2024)

Figure 8 – Shock of Durable Sector productivity to Output, Wages, and Inflation - A^D



Source: Author's calculations (2024)

3.5 SENSIBILITY AND IDENTIFICATION ANALYSIS OF KANSAI-RS MODEL

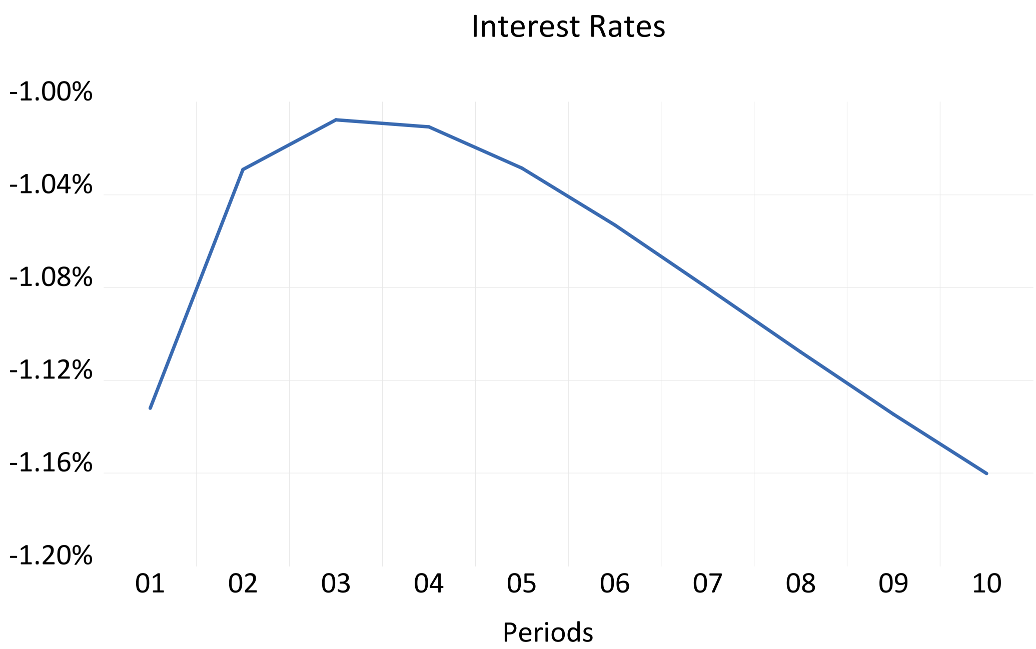
The Kansai-RS Model is sensitive to some parameters. For example, the parameters related to labor demand can modify the model results in such a way that the model explains the RS cycle in an opposite way. This is because if those parameters make labor reacts differently to productivity shocks, output might have unsatisfactorily outcomes. Also, parameters related to the New-Keynesian Philips Curve (NKPC), such as the Calvo Lottery parameter, and the proportion of Ricardians and Non-Ricardians (Hand-to-Mouth, or HTM consumers) in the model, modify the results. Therefore, hundreds of simulations were done in order to capture the best possible the cycles of output, total worker's income (W) and inflation.

The Frisch elasticity parameter and the disutility of labor for switching sectors were tried around the national and international literature values (which are 1.94 and 1.5) until the model captured relatively well the cycle of output and remaining series. Also, the autoregressive components of the exogenous series change the model dynamics. Since data is in annual frequency, the best option was to set it to zero if it was not significant in the estimation using the cycle series.

Another fact noticed is the cyclical behavior of output and inflation in the model, depending on the NKPC and parameters related to it. If NKPC parameters change, the correlation between inflation and output can be positive or negative, which changes the model dynamics during the shocks. For future research, the use of estimation techniques instead of calibration might improve the model's performance.

In relation to the monetary policy rule, a curious model behavior was identified if there are only local government shocks. Since the model's Taylor Rule captures the changes in regional inflation and output gap, if the regional economy is affected by a sequence of downward local government expenditure, the monetary policy shall respond with decreases in interest rates. However, since interest rates are set based on the national economy, they might not decrease as much as the model shows. Therefore, for a sequence of local government policies, it shall be better to reduce the sensibility of the monetary policy to output gap and inflation. Graphs 9 shows what happens in a sequence of decreases in local government spending and investment if the Taylor Rule is highly sensitive to inflation and output gap. In summary, interest rates can decrease if the local government keeps a constant austerity policy, and perhaps it shall have a smaller reaction since interest rates are set based on the national economy and not strictly to the RS economy.

Figure 9 – Simulated Interest Rates after a sequence of only local government spending/investment decrease



Source: Author's calculations (2024)

3.6 CONCLUSION

The current study aimed to apply a regional DSGE model for the State of Rio Grande do Sul (RS), named Kansai-RS model. Since this specific Brazilian state is one (if not the most) of the states facing a weak fiscal condition, it is reasonable to try to identify if this type of methodology might help to understand the regional economy and how economic policies can help in its growth and development. After the exposure of the model, based on Okano et al. (2015), we made some adaptations. For example, central government spending was included in GDP, as well as public investment. Also, a whole section about a parametrization for the State of RS is widely shown, time series constructed for productivity, an estimation of a Taylor's Rule with regional data and the estimation of cycles and shocks using econometric techniques are done as well.

Some simulations showed interesting results for the Kansai-RS model. Although not all results matched with real data moments, it could capture some real-world dynamics of RS and can be a starting point to be improved and used to explain better this Brazilian region. For example, local government spending shocks are positively correlated with output growth and wages, such as real data shows. Also, productivity shocks are negatively correlated to local inflation, which occurs for RS data too. On the other side, some model moments are not attached to the data moments. For example, central government spending shock is negatively correlated to RS output, according to the sample period of the data, but the model shows a procyclical behavior. Also, some standard-deviation moments are higher than those from data. Therefore, the model is reasonable to explain some variables for some shocks, and not much for others.

Two points are important for now on: using this model as a starting point to simulate other types of shocks for the next years, such as the payment of RS debt owned against the central government and explored in Essay 4, and to verify possible variables paths for the future. Also, since the model captures well the dynamics of productivity shocks on output, the negative impacts of climatic events on output, such as droughts or intense periods of rain/floods, such the ones that happened in 2024 in dozens of cities in

RS, can be studied[16].

For future research, there are some points already clear from the results of Kansai-RS. To improve the model, including the agribusiness sector in the model, or switching it with the durable sector that represents civil construction, might show better results for the simulations, since RS economy is even more dependent on this type of activity (agribusiness). Secondly, using data for Bayesian estimation might improve the parameters setting as well as the model results. Thirdly, improving the sample periods of the series and turning the model to quarterly frequency might be another agenda for future research. Lastly, other improvements can be done to the model equations, not only for firms, but also for households (preferences) and government (further division to incorporate public employees wages, following Papageorgiou and Kazanas (2013) for example). However, the results are positive and make the Kansai-RS model already able to be used for simulations for the RS economy.

[16] Lindé (2018) brings the possibility to use climatic events shocks in a DSGE model.

4 THE FISCAL POLICY OF RIO GRANDE DO SUL: SUBNATIONAL PUBLIC FINANCE ANALYSIS AND SIMULATIONS FOR THE FISCAL RECOVERY REGIME USING KANSAI-RS DSGE MODEL

The following essay is an applied study of the Kansai-RS model to simulate the RS economy during the years of the Fiscal Recovery Regime, as well as providing a contextualization of the past and current situation of the most indebted States in Brazil and, mainly, Rio Grande do Sul.

4.1 INTRODUCTION

The present essay aims to address what shall happen in the next few years to the State of Rio Grande do Sul (RS) economy in light of the beginning of debt payments to the Brazilian central (or federal) government. In order to pay the State debt, RS local government assigned an agreement named the "Fiscal Recovery Regime", which outlines the probable debt installment values to be paid over the years. This topic is central for the public finance debate of RS and other states in Brazil, such as Rio de Janeiro, Goiás, and Minas Gerais.

Other studies, such as Plosser (1989) and, Fochezatto (2003) have used Computable General Equilibrium (CGE) models to study the RS economy and how shocks affect its economic dynamics. Now, using the Kansai-RS DSGE model, new outcomes can emerge to enrich the debate due to the different methodology. It is expected that the state's public debt payment might significantly reduce RS output growth, since a significant part of local output consists of State government spending (around 7% of output). Therefore, this essay is presented as follows.

In section 4.2, a contextualization of the subnational public finance, or, in other words, the Brazilian states' public finance situation, will be exposed, as well as its origins decades ago. Later, section 4.2.1 will present the specific RS perspective, which is one of the weakest fiscal positions in Brazil (if not the weakest). Section 4.2.2 discusses the evolution of RS debt alongside the Fiscal Recovery Regime. Finally, simulations using the Kansai-RS DSGE model will demonstrate how the RS economy shall behave during the FRR. Therefore, these simulations can provide evidence of a possible necessity for renegotiation of the RS public debt, if the regional economy suffers output losses during the debt payment. Moreover, since the RS historical perspective and the model simulations can provide relevant information for this debate, they are one possible justification for the existence of this present work.

Regarding the research problems, the desired questions to be answered are the following:

1. How did the indebted Brazilian states arrived at the current situation?
2. How is RS in relation to these states? Is it better or not?
3. Using Kansai-RS model, what are the perspectives for the RS economy during the FRR?

The main hypothesis is that the RS local government will have a relevant part of its tax revenue drained, and, since it is a very weak state in terms of public finance, it has less margin for tax hikes or spending/investment cuts. Thus, the loss of regional output might be significant during the years of the FRR plan. In Section 4.3, three types of simulations will be done: i) Local government spending cut; ii) Local government investment cut; and, iii) Local government tax hikes on consumption, which is one of the major revenue sources of state governments in Brazil.

4.2 SUBNATIONAL PUBLIC FINANCE IN BRAZIL

The public financial crisis in subnational entities of Brazil is not recent. Lopreato (2002a, page 62) and Lopreato (2002b) posit that its origins date back decades, around the 1970s. Half a century has passed since then, and past and present proposed solutions are far from their conclusions, while several states still owe significant amounts of debt to the Brazilian federal government. Focusing on the highest state GDP in Brazil (São Paulo, Rio de Janeiro, Minas Gerais and Rio Grande do Sul), Lopreato (2000, page 135) shows that, until the 1990s, most of the state debt belonged to these states, accounting for around of 90% of the total. In the 2020s, all states together broke the record of 1 trillion of Reais of debt (NTB, 2024b). This scenario, which has been present for decades, only shows the need to research deeper into the national public finance, analyzing the subnational entities as well. One example is Vasconcelos (1999), who presents the subnational finance perspective for three Brazilian states, which are Ceará, Rio Grande do Sul and Paraná, comparing their results to each other and showing how Ceará conducted its fiscal adjustment in the late 1980s.

To overcome this century-old problem of subnational public finance crisis, the federal government of Brazil proposed a regime to initiate the debt installments payments gradually, until the states can pay the full amount of each debt installment again (in 2017, the Supreme Court of Brazil suspended the payment of debt installments until 2023). This new regime is named the "Fiscal Recovery Regime", and it presents a schedule extending until 2031 for the states to pay the full amount of their debt installments and finally start to successfully pay their total stock of debt. Therefore, the subnational finance weakness, which began decades ago, will persist for some more decades.

During the 1970s and 1980s, some economic conditions helped the states to be indebted but not suffer as much as in the 2000s onwards. The 1970s experienced greater GDP growth in Brazil, alongside increasing inflation. While the years of 1980 did not present much economic growth, inflation reached high levels, leading to a hyperinflationary process. This created inflation tax not only for the federal government, but for the subnational governments as well. This effect is usually called "Tanzi Effect", when high inflation depreciates government costs and, for example, when wage payment is made, it has less real value.

Another point in relation to government revenue is that, until the years of 1990, state governments could also issue bonds, so a primary deficit could still be financed by their own debt. Moreover, the years of 1990 marked a period of privatization of public companies (both federal and state), which helped to maintain the health of state public finance for a while, with increased extraordinary revenue. However, after the end of the 1990s, when there was (almost) no more public assets to sell, inflation tax (since the Real Plan stabilized inflation after 1994) or bonds for the state governments to finance themselves (SANTOS, 2022), the only way to solve the state debts was fiscal discipline in order to achieve a government primary surplus[1]. Furthermore, the government primary surplus should be enough to cover all interest payment on the stock of debt, and the amortization required. For example, in recent years, this has not happened for Rio Grande do Sul's public finance in almost any year.

4.2.1 The State Finances of Rio Grande do Sul

The State of Rio Grande do Sul (RS) faces the same fiscal problems as other aforementioned Brazilian states in Section 4.2. According to IFI (2020), this specific state is the weakest one in terms of public fiscal indicators. Also, other studies such as Junior (2005), Júnior and Jacinto (2011), Nova and Marquetti (2009) and Junior (2015) provide an extensive analysis of the historical perspective of public deficits and fiscal sustainability of RS, dividing the analysis by sample periods (1970 to 2003, 1970-1997 and 1997-2003, 1999-2006 and 2011-2014, respectively). Moreover, Junior (2005) and Nova and Marquetti (2009) show econometric studies to test, for example, if long-run public finance sustainability holds in RS and under what circumstances, and what may cause the fiscal deficits to occur constantly. Hence, a vast literature has already explored this specif subnational entity over the last decades.

To summarize the reasons for constant public deficits, it is important to understand the current economic conditions of federal unity in Brazil. As shown in subsection 4.2, when a subnational entity cannot issue money or debt, and has limited possibilities for public asset selling and no other extraordinary revenue source such as inflationary tax, it must have less government expenditure than revenues (taxes) to pay its debt (interest on

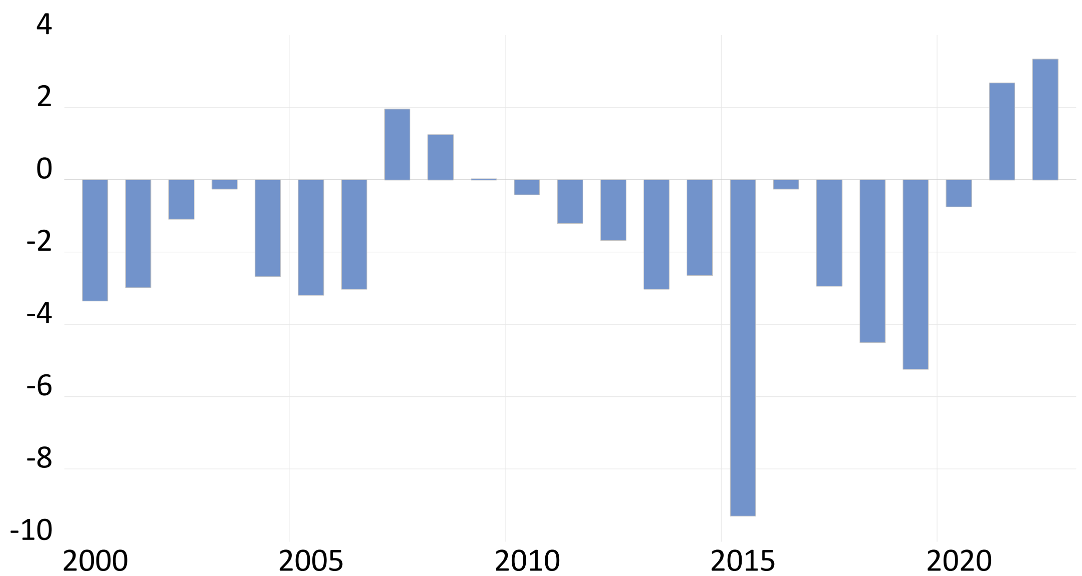
[1] For example, many public state banks were sold in the 1990s and 2000s. This fact helped to lower the state's debt for a while. A greater discussion is presented on Brandão (2009) and Luz and Videira (2013). However, it is important to highlight that Rio Grande do Sul was one of the few states not to sell its public bank. This fact leads to intense debates during every electoral period, when the proposal to sell this asset to lower the debt or make regional investments is occasionally brought up

the stock of debt and amortization) (SANTOS, 2022). Recently, the RS government sold some remaining public companies in 2021 and 2022, which created one of the few nominal surpluses during the last 50 years. Figure 10 shows the nominal results from 2001 to 2022 in real terms. It is possible to notice that, except for the years 2021 and 2022, only 2007, 2008 and 2009 also presented positive nominal result, with 2009 being almost even.

There are some main factors that frequently create nominal deficits in the RS public budget. They are the following:

1. High share of state expenditure on public employees wages;
2. High share of state expenditure for retired employees' wages and other pensions;
3. High share of state expenditure in debt amortization and interest;
4. Significant financial deficit related to the retired employees wages;
5. Cyclical climatic events that reduce the state GDP, reducing tax revenues.

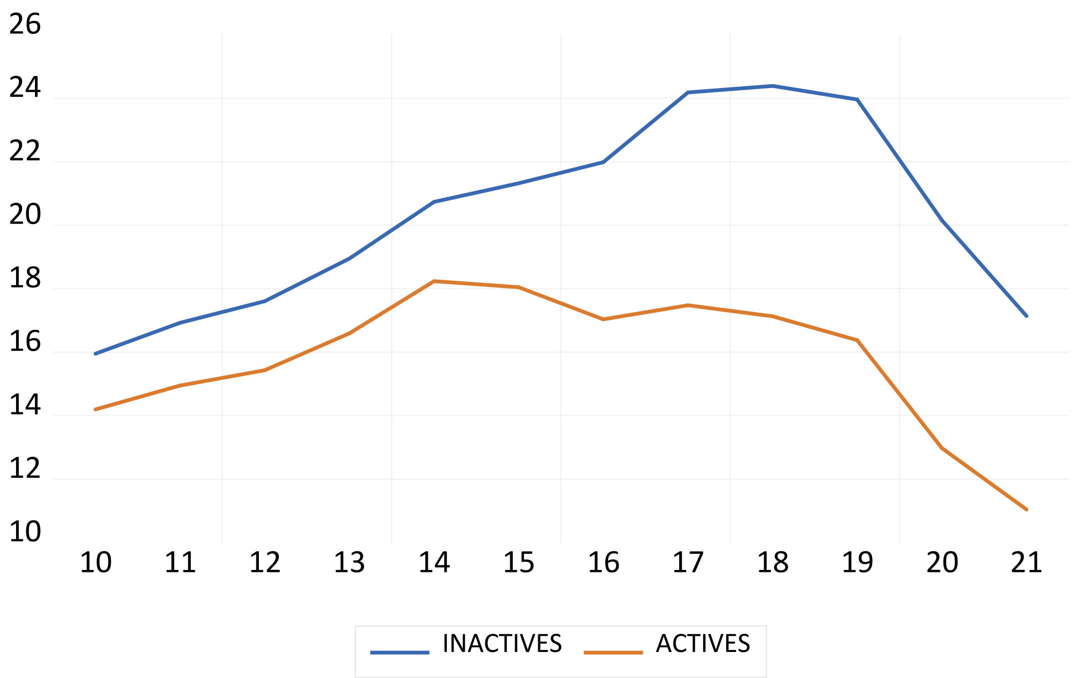
Figure 10 – Nominal Result of RS in Billions of Reais — Prices of 2022 (Deflated by IGP-DI)



Source: SEFAZ/RS (2022a)

For the first and second items of the list, it is possible to note that these specific expenditures increase rapidly over time. However, the second one (expenditure on inactive employees) increases even faster (Figure 11). Although both are decreasing in real terms, they consume a significant proportion of local revenues. In 2022, for example, together they represented around of 60% of the total amount of RS expenditure (SEFAZ/RS, 2022b). Also, the public employees' contribution for retirement was insufficient in the last decades (and still is), thus creating significant pensions deficits each year (item 4).

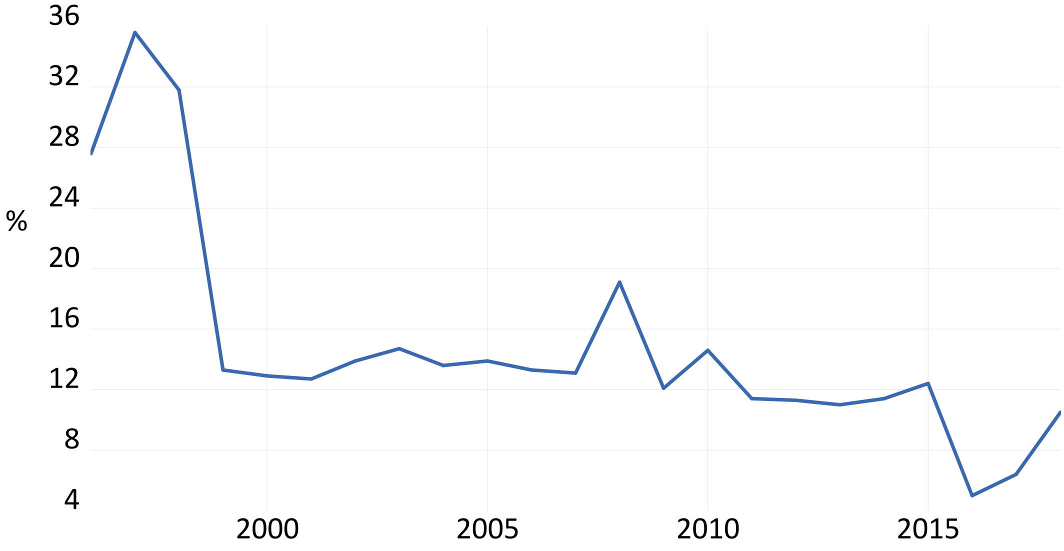
Figure 11 – RS active and non-active employees expenditure in Billions of Reais -Prices of 2021 (Deflated by IGP-DI)



Source: SEFAZ/RS (2022d)

In Figure 12, it is possible to verify that, since the late 1990s, RS has allocated around of 12.5% of its net revenue to debt interest and amortization payments. Summing with active and non-active employees expenses, RS spends around of 90% of the state net revenue with those expenses. Therefore, the State of RS has limited possibilities to boost economic growth through public policies such improvements in the health system, education, infrastructure, or public security.

Figure 12 – Debt Cost over State Net Revenue (in %)



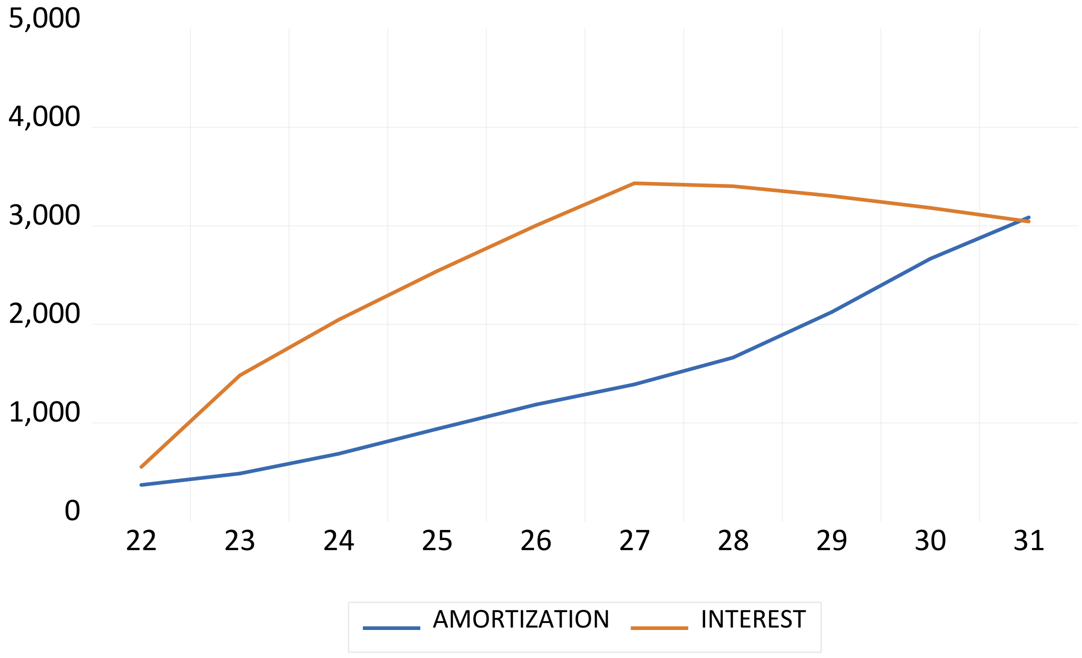
Source: SEFAZ/RS (2022c)

4.2.2 Evolution and Negotiation of Rio Grande do Sul Public Debt and the current Fiscal Recovery Regime (FRR)

The current plan for RS and some other Brazilian states that are indebted to the federal government is named the Fiscal Recovery Regime (FRR), which goes from 2022 to 2031. The states that are under this new regime have a payment plan to restart their debt amortization, reaching a full installment payment in 2031. From 2022 to 2031, these states will pay from 0% of their debt installment to 100%, increasing by 11% each year. For example, if a state has a debt payment plan of 1 billion Reais starting in 2022, it will pay 110 million Reais in the year of 2023, 220 million Reais in 2024, and so on, until the full amount in 2031. In 2022, there is a 100% payment suspension and the non-paid value for each year will be incorporated into the stock of debt. SEFAZ/RS (2024a) and NTB (2024a) present a detailed schedule and economic forecasts of GDP and debt installment amounts for each year of the FRR.

In terms of the formulation of the FRR, a researcher might ask why it was conceived this way, and the reason is quite simple. Due to the fact that both state (local) and federal (central) governments project that economic activity will increase in the following years (after FRR began in 2022), government revenue from taxes shall increase as well. However, if some other event occurs, such as a drought, flood or a period of intense rains in RS and GDP falls, the State government might not be able to honor its agreement. Also, not only can downward economic activity frustrate the FRR plan, but the debt indexing might also do this as well. The FRR sets the interest payment on debt as the Brazilian official consumer price index, named IPCA, plus 4%, or the Brazilian basic interest rate, named Selic. The lower rate will be chosen. Hence, if interest or inflation rises, the interest on debt will increase. Also, there is another negative possibility, which is when economic activity falls and the debt index rises. For instance, this scenario is the worst one possible. Will the RS government be able to pay the debt installment correctly every year? It might depend on economic activity, as well as the behavior of the debt index. The DSGE model used for RS in essay 3, named the Kansai-RS model, will be used in the next section to verify the path of macroeconomic variables during the FRR period and beyond.

Figure 13 – Forecast for amortization and interest on RS debt in Billions of Reais — Prices of 2022 (Deflated by IGP-DI forecasts)



Source: SEFAZ/RS (2024b)

Figure 13 shows the path of future payment in terms of amortization and interest on debt. According to the Secretary of Finance of RS(SEFAZ/RS, 2024a), which made available forecasts for these two expenses until 2031, it is possible to see that they will reach about ten billion Reais by the end of the forecast. This scenario, combined with the increasing spending on active and non-active employees, shows that RS government will have much fewer possibilities for output growth and public policies than in the beginning of the FRR. Simulations using a dynamic macroeconomic model in the next section will explore some scenarios, such as ranging from a high output growth to a low or negative growth and a higher payment for interest on debt scenario. The values presented in Graph 13 will be normalized to the RS output and used as data for the simulation shocks. The objective is to analyze what shall happen when the RS government pays debt installments and interest over debt in the following years, withdrawing revenue usually used of other purposes.

4.3 SIMULATIONS FOR THE FISCAL RECOVERY REGIME OF RIO GRANDE DO SUL

In this section, five simulations using Kansai-RS model will be performed:

1. Local debt payment without any other shock hitting the economy;
2. Local debt payment with negative productivity shocks hitting the economy;
3. Local debt payment with positive productivity shocks hitting the economy;
4. Local debt payment with interest rates shocks increasing the interest payment on debt and negative productivity shocks hitting the economy;
5. Local debt payment with interest rates shocks increasing the interest payment on debt and positive productivity shocks hitting the economy.

This will make it possible to visualize what shall happen when the local government starts paying its debt owed to the central government. Also, since each year the debt installments increase, due to the 10-year period until the full amount of amortization is paid, according to the FRR, it will be possible to visualize the path of output in those scenarios, followed by concluding remarks.

According to Section 3.5, it was identified that the monetary policy rule changes output dynamics in the presence of a sequence of local government shocks. Therefore, it is necessary to offset an undesired reaction of interest rates due to a local output decrease. This could lead to a decrease in interest rates and create the behavior of a growing economy, which is probably not the case during the FRR.

The equation 47 now has a new component, which is a shock draining revenue from the local government budget constraint. It is as follows:

$$G_t^L + IG_t^L = (1-\theta_C)\left(\frac{\tau_t^C P_t^C C_t + \tau_t^D P_t^D I_t^D}{P_t}\right) + (1-\theta_W)(\tau_t^{WC} w_t^C N_t^C + \tau_t^{WD} w_t^D N_t^D) + (1-\theta_F)\tau^F(\phi_t^C + \phi_t^D) + \omega TR_t - \epsilon^{BL}. \quad (55)$$

In order to conduct this type of simulation, to address the trajectory of output cycle during the local government spending/investment reduction, ϵ^{BL} will be incorporated to the model in two ways[2]:

$$G_t^L = \rho_{G_t^L} - \epsilon_t^{BL}, \quad (56)$$

or

$$IG_t^L = \rho_{IG_t^L} - \epsilon_t^{BL}, \quad (57)$$

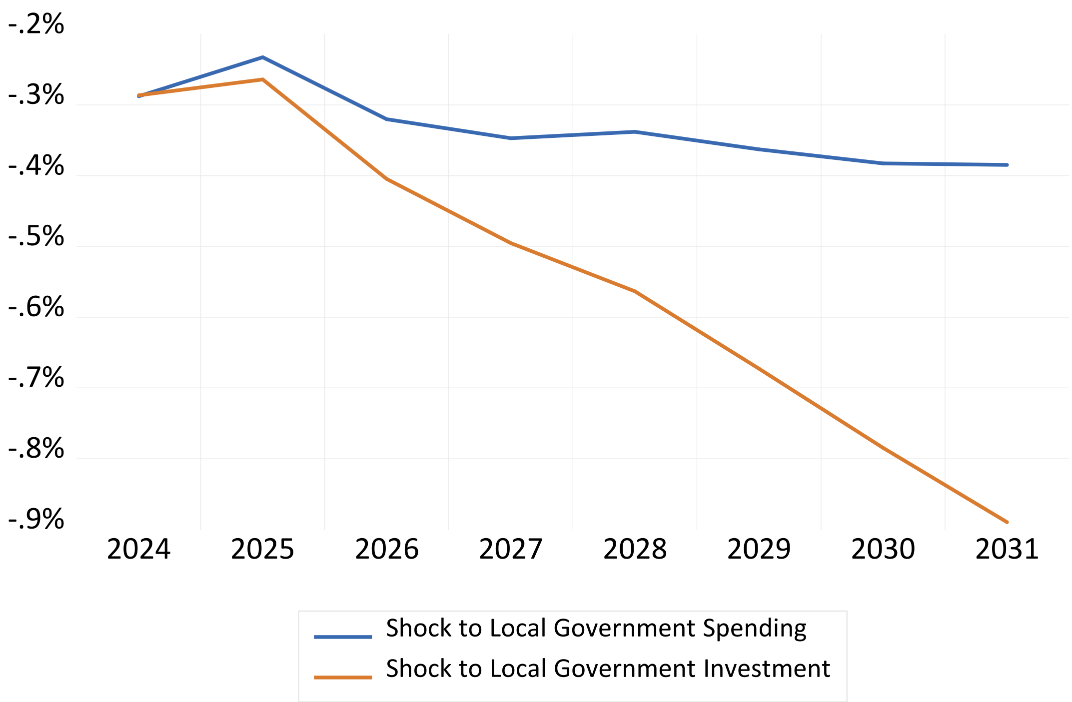
where ϵ_t^{BL} refers to a negative revenue shock from the payment of local bonds in relation to amortization and interest. The size of the shock is proportional to the steady state of G^L or IG^L. Thus, ϵ_t^{BL} for IG^L must be larger, since the steady state of IG^L is smaller than the steady state of G^L[3].

[2] Remembering that θ_F is equal to 1, since all firms' tax revenues go to the central government budget. States and municipalities in Brazil receive taxes mainly from consumption, such as ICMS (Tax on consumption of goods and services) and ISSQN (Tax on services of any nature).

[3] For example, if amortization and interest over debt are 1% of RS output in a specific year, it will enter as $\frac{\epsilon_t^{BL}}{G^L}$ or $\frac{\epsilon_t^{BL}}{IG^L}$. Since G^L is 0.113 of output and IG^L is 0.0137 of output, $\frac{\epsilon_t^{BL}}{G^L} = 0.0884$, while $\frac{\epsilon_t^{BL}}{IG^L} = 0.7299$ of output.

The following graphs show the behavior of output in relation to its steady-state ($\hat{Y}$) during the scenarios, where the debt payments will come from negative shocks to local government spending or investment (i.e., reductions in public spending or investment). In the absence of productivity shocks, the debt installments cause GDP to fall. As the debt installments increased, output declined further (Graph 14). This fact is in line with the previous planning between the local and central governments regarding the RS debt, where the debt installments would increase according to the growth of RS economy. Nonetheless, if a negative shock hits the economy, especially a productivity one, the planning fails, since output enters a recession, and, consequently, taxes fall as well (Graph 15). In the presence of positive periodicity shocks, RS economy performs well and the local government can pay the debt installments without a recession (Graph 16).

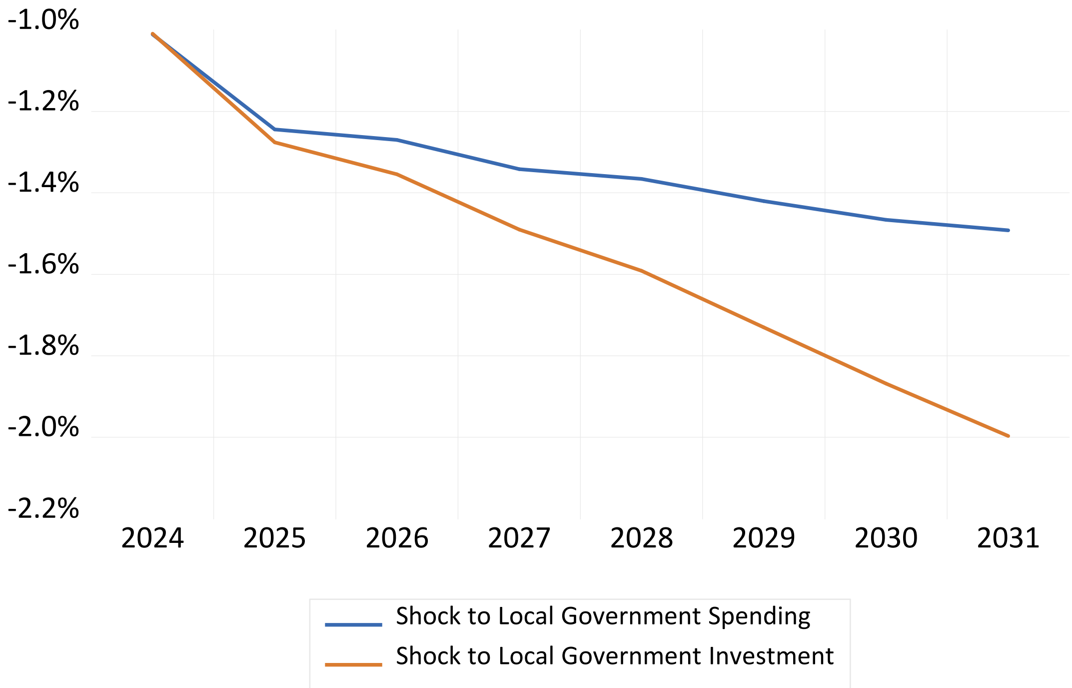
Figure 14 – Scenario 1 for Output Cycle ($\hat{Y}$): No other shock hitting the economy — 2024 to 2031



Source: Author's calculations (2024)

About the fourth and fifth scenarios (Graphs 17 and 18), where interest on debt increases or decreases, it was interesting to notice that, in the presence of productivity

Figure 15 – Scenario 2 for Output Cycle ($\hat{Y}$): Negative productivity shocks hitting the economy - 2024 to 2031

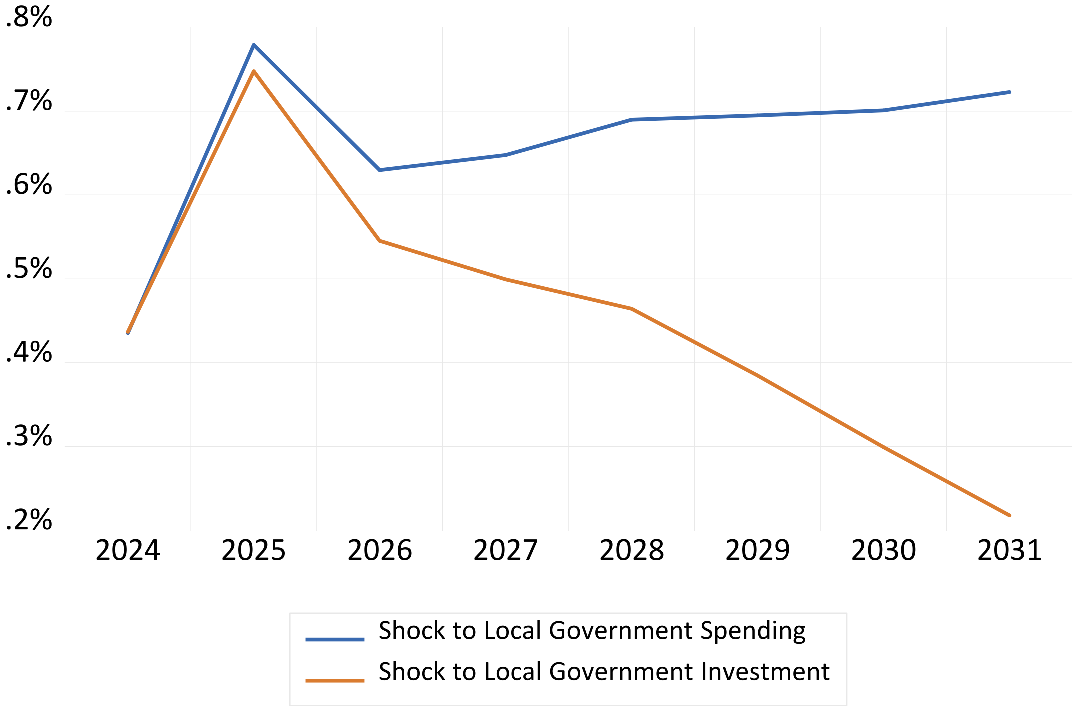


Source: Author's calculations (2024)

shocks (negative or positive ones of 1%), the payment of interest on debt and amortizations does not affect the RS economy significantly. Therefore, interest rate shocks were not significant enough to change output dynamics in the model. These simulations show the importance of positive productivity shocks, making the RS economy grow. If the regional economy does not grow in terms of output, the debt installments will consume tax revenue from the local government leading to a natural decrease in GDP, since the local government has less impact on output.

One positive aspect of the FRR simulation to be highlighted is the dependence of the RS cycle on productivity shocks, which aligns with Table 17, where the majority of output variance reacts to productivity shocks. This fact is also noticed for Brazilian and international models, but it seems in more intense for RS. Droughts, floods, intense rain, or heat may explain this significant behavior of productivity shocks on output, since climatic events often drop RS economic activity.

Figure 16 – Scenario 3 for Output Cycle ($\hat{Y}$): Positive productivity shocks - 2024 to 2031



Source: Author's calculations (2024)

Finally, in line with Paiva (2019), local public investment appears to be much more important than local public spending for output dynamics. This highlights the importance of adequate local public policy to ensure the growth and development of a region.

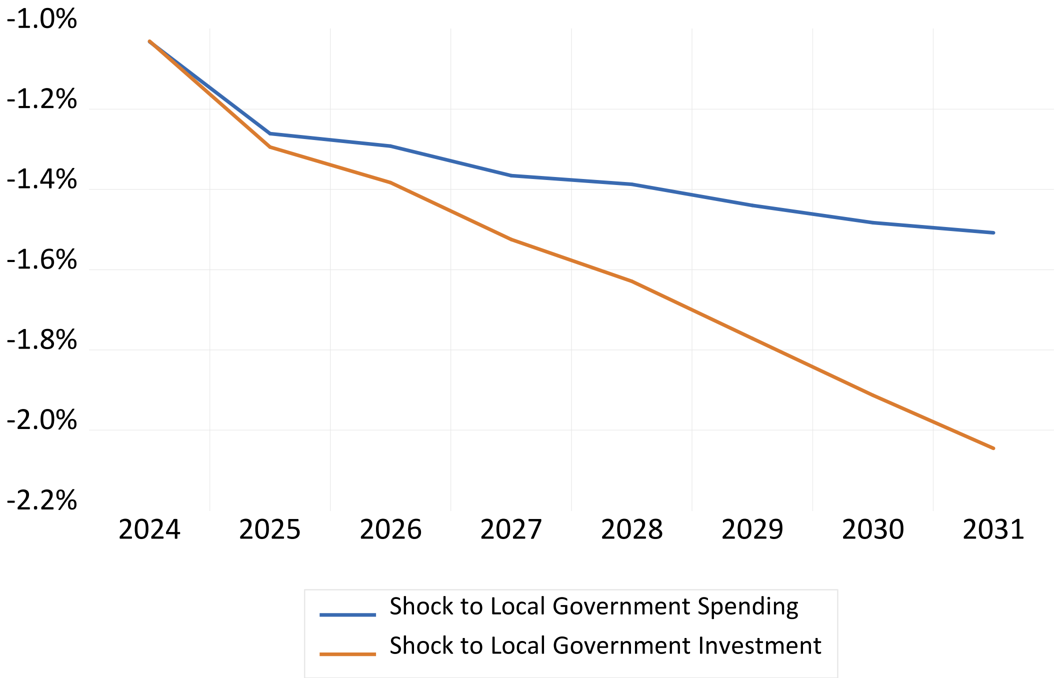
4.3.1 What if the Local Government increases taxes to maintain its spending/investment policies?

Now, a different simulation is conducted. In order to maintain government spending and investment, the local government will increase taxes on consumption, since this tax is one of its major sources of revenues[4]. Now, the shock will be incorporated into the following equation:

$$\tau_t^C = \rho_{\tau_t^C} + \epsilon_\tau^C. \tag{58}$$

[4] In Brazil, all states can set their tax rate on consumption, named ICMS (Taxes on goods and services). It is a major source of tax revenue. In the beginning of March 2024, the RS government was trying to increase this tax rate in order to keep its budget balanced with the payment of debt installments under the FRR and the restructuring of public employee careers.

Figure 17 – Scenario 4 for Output Cycle ($\hat{Y}$): Negative productivity and increasing interest shocks - 2024 to 2031

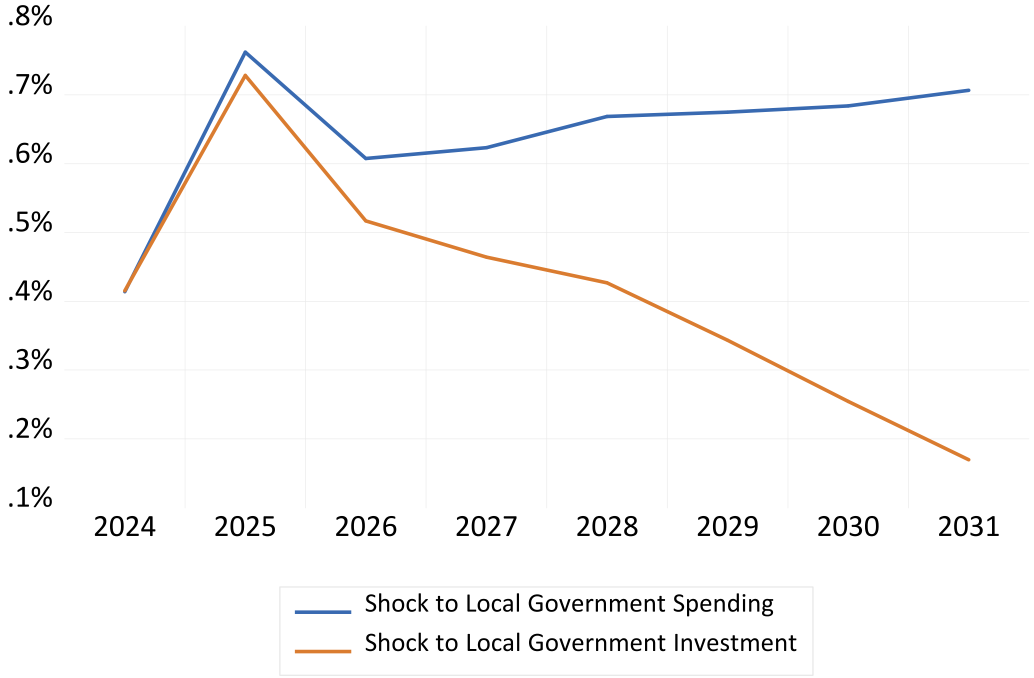


Source: Author's calculations (2024)

The tax increases are set in accordance to the debt installments values, in order to maintain G_t^L and IG_t^L unchanged, with no need of reduction. Alesina (2019) and Alesina, Favero and Giavazzi (2019) show that a fiscal adjustment by spending cut is better in terms of GDP loss than through taxes increases. After this type of simulation for RS, it will be possible to notice if Kansai-RS is in line with these works.

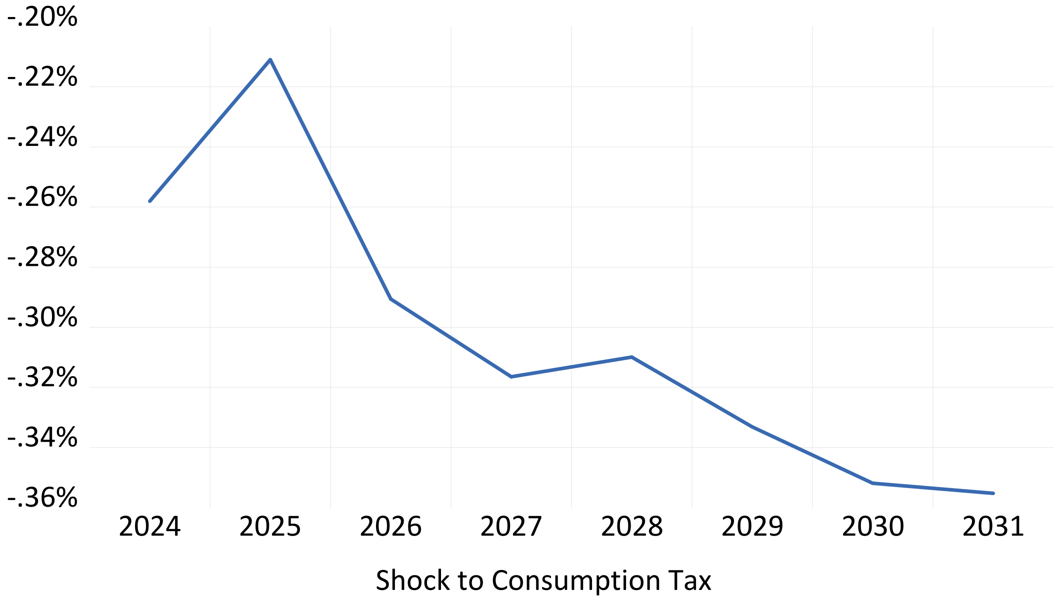
Visualizing Graphs 19, 20 and 21, it is possible to see that tax increases create a similar dynamic to spending cuts. If no shock hits the economy and taxes rise in order to pay the FRR debt installments, output falls. If positive productivity shocks hit the economy, output slightly increases. For a negative productivity shocks scenario, output falls. Therefore, the worst scenario among the three types of simulations (government spending cut, government investment cut or tax increases) is when local government cuts public investment.

Figure 18 – Scenario 5 for Output Cycle ($\hat{Y}$): Positive productivity and increasing interest shocks - 2024 to 2031



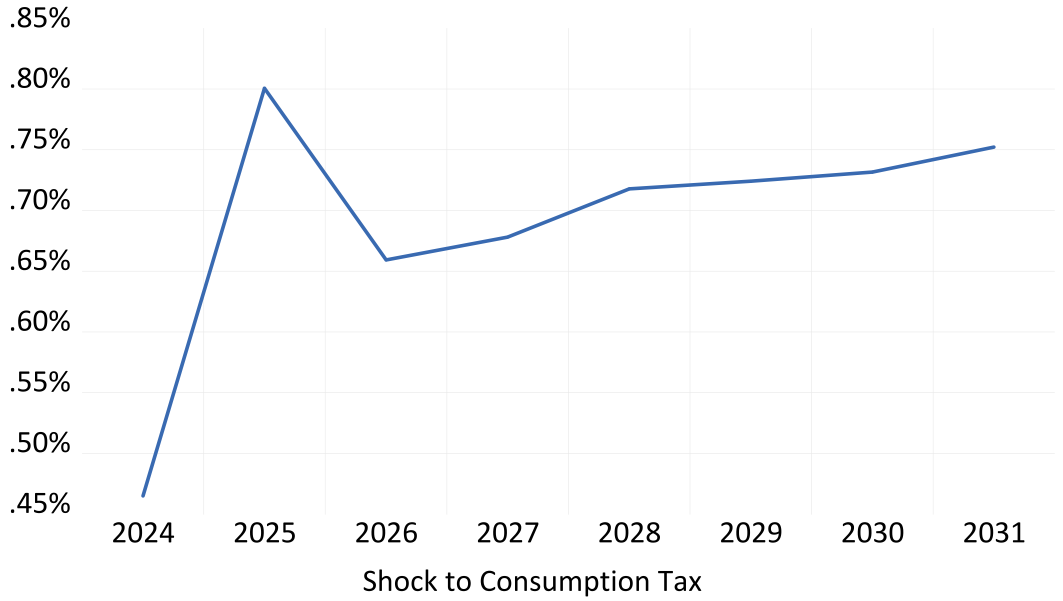
Source: Author's calculations (2024)

Figure 19 – Output cycle path ($\hat{Y}$) in the presence of tax increases and no other shock hitting the economy - 2024 to 2031



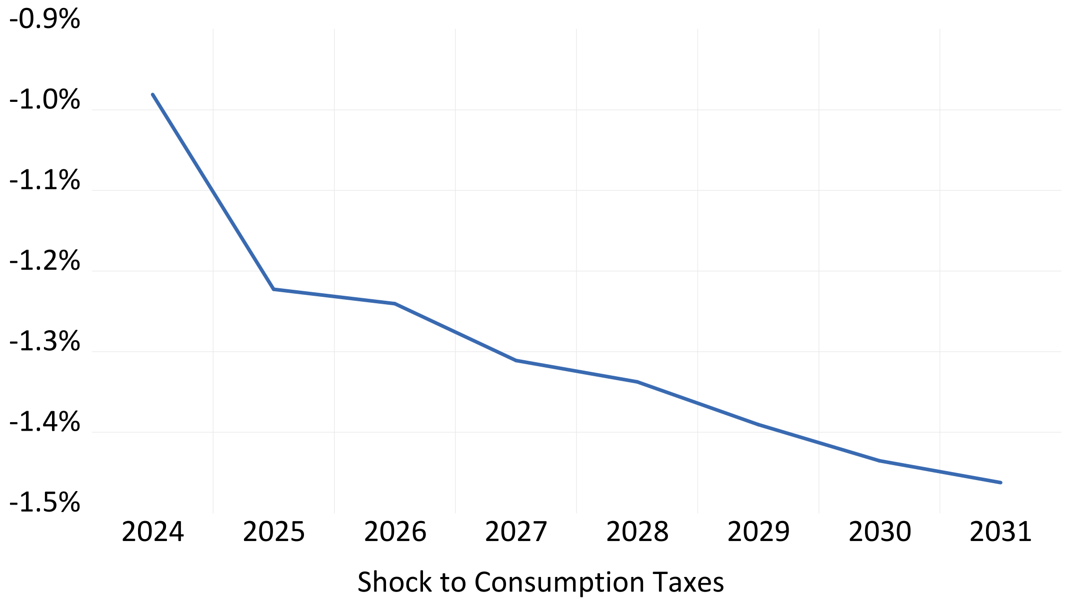
Source: Author's calculations (2024)

Figure 20 – Output cycle path ($\hat{Y}$) in the presence of tax increases and positive productivity shocks hitting the economy - 2024 to 2031



Source: Author's calculations (2024)

Figure 21 – Output cycle path ($\hat{Y}$) in the presence of tax increases and negative productivity shocks hitting the economy - 2024 to 2031



Source: Author's calculations (2024)

4.4 CONCLUSION

The State of Rio Grande do Sul (RS) is facing a challenging situation, and this is not recent. Since the 90s, the state stock of debt has been increasing in a non-payable way, along with other concerns such as high spending on active and retired public employees and other types of pensions for households (related to public employees). Also, RS State is aging faster than all other Brazilian states, which suggests that this type of expenditure might increase in the coming years, as well as for public health.

In 2021, RS signed an agreement with the central government of Brazil to start paying its debt. However, the payment would not be in its full value since the beginning: the debt installment would increase during the years from 2023 to 2031, with the hope that output would increase and boost tax revenues. After all, the debt installments are a draining of revenue from the local government budget that would probably decrease output. Using the Kansai-RS DSGE model, the main objective of the current study was to study the possible trajectories of the regional output during the years from 2024 to 2031 under the FRR, and the results were not so optimistic.

In one scenario, where no shock increased/decreased interest payments over debt nor productivity, the debt payment already caused GDP to fall. However, in the presence of a small positive shock of productivity, the RS economy could grow instead of the local revenues being drained. Also, if negative shocks of productivity hit the economy, the scenario is not good. The RS economy decreases, emphasizing the need for public policies to stimulate a sustainable growth of the regional macroeconomy, even more in the presence of adverse climatic events. Also, since the Kansai-RS model shows that RS GDP suffers more with public investment cuts than public spending cuts, the results are in line with Papageorgiou and Kazanas (2013), who claim that a public policy to boost public investment is more efficient than boosting public spending. Also, Paiva (2019) posits that public investment boosts output more than other types of spending, and Kansai-RS model simulations are in line with him as well.

Regarding tax increases to maintain government spending and investment, output presented similar dynamics to government spending cuts. It is interesting to highlight that Junior, Teixeira and Silva (2022) found the results of a tax decrease for one specific state of Brazil (Goiás) in relation to all other states. If Goiás decreased consumption tax, its output grew. However, if every other state decreased consumption taxes as well, Goiás output fell. This is in line with the Kansai-RS model, where a tax hike decreased output.

The scenarios in Section 4.3 explain and demonstrate what might happen in the RS economy from 2024 to 2031 during the FRR regime. Productivity shocks demonstrated to be fundamental for the sustainable growth of RS, and the need to rethink the FRR payment during negative shocks is also a concern. For example, at the beginning of May 2024, a huge flood affected dozens of cities, including the capital of RS, Porto Alegre. This led immediately the Brazilian central (federal) government to suspend the debt installments for three years, as well as the incidence of interest over debt during this period.

On the other hand, it is also possible to state that an improved model can explain and demonstrate even further the consequences and possible outcomes for the next years in the RS economy. This is not strict to the model, but also for the available data for RS. Alongside the inclusion of the agribusiness sector in the model, expanding the series samples and/or turning it to quarterly frequency might improve this type of analysis for the RS economy.

In short, according to Kansai-RS results and in accordance with economic theory, the best possibilities for the RS State, which has a worse fiscal situation in relation to other indebted states (item 2), are to use economic policies to boost productivity growth. Among three options to pay its debt owed to the central government, the best is to reduce spending or raise taxes, and the worst scenario is to cut investment. However, it is important to highlight that tax hikes can be worse for economic growth since they reduce consumption. Hence, tax hikes could be bigger than predicted to keep government spending and investment constant. Nevertheless, cutting public investment is much more harmful than cutting spending. If the FRR has other options for agreements, paying the debt in another way, such as public estate transfers (public companies and properties) to the central government, might be part of the solution. Also, reducing interest over debt is important, as well as a clause that pauses the payment and does not run interest while it is paused in case of natural disaster, national recession, or other factors that induce GDP to fall. Ideally, as less as possible output suffer and more quickly the end of the state debt comes, the better for RS, responding to the research problem in item 3.

5 CONCLUDING REMARKS

The present Thesis aimed to study regional DSGE models, a field within the DSGE methodology that is increasingly growing, as shown in Essay 1 (content 2). This type of DSGE model opens new possibilities: a researcher can apply a DSGE model to a specific region, which can be small in terms of output, geographic size, or both. Developed countries, such as Japan, and developing countries, such as Brazil, are "divided" into many regions. Therefore, the possibility of creating a specific model for each region arises. Table 1 shows many models applied to regions around the world, as well as their main characteristics, results, and lessons.

One possibility, alongside surveying these models, was to apply one of them to a specific region. This is what Essay 2 (content 3) did, using the State of Rio Grande do Sul (RS) for this study. Based on the model of Okano et al. (2015), a regional model was parameterized and adapted for the RS economy, especially in the government block of equations. The results were positive, although further research agenda and possible limitations were also identified. In general, the second moments (correlation and standard deviation) presented positive results for some variables. Additionally, one-time exogenous shocks presented reasonable results, as did the impulse-response functions. Beyond the one-shock instance, one type of simulation analyzed a sequence of shocks, estimated with RS data using simple econometric techniques (OLS), and the results were also positive. Some other parameters were also estimated, as shown in Appendix B, and they presented interesting results, such as demonstrating that regional data might explain interest rate settings by a national government authority.

Essay 3 (content 4) used Kansai-RS model to simulate a future sequence of shocks, starting in 2024, due to the debt installments of RS local government. It was possible to notice that productivity shocks have a crucial influence on RS output, and in the absence of positive productivity shocks, the RS economy might suffer during the FRR regime. For example, reducing public investment to pay the debt installments was the least favorable option to be used in terms of output growth.

The results of the Kansai-RS model were also in line with other works, such as Papageorgiou and Kazanas (2013), Paiva (2019) and Junior (2015), which focused more on fiscal policy simulations. Papageorgiou and Kazanas (2013) and Paiva (2019) found that public investment has more positive effects on output than other types of local expenditures. Moreover, Junior, Teixeira and Silva (2022) found that a decrease in consumption taxes can be beneficial for output growth, and the Kansai-RS model showed that consumption tax hikes are harmful. Therefore, the way how the RS government will pay its debt presented to be highly relevant. About productivity dynamics, Okano et al. (2015) showed that regional differences are relevant for output growth, and Kansai-RS model results verified that RS economy is also very sensitive to adverse productivity shocks. Hence, this type of concern must be part of the governmental agenda for the next few years.

For future research, there are many possibilities. Firstly, the regional model for RS can add the agribusiness sector and become an open economy (interacting with other states and countries). For households, other types of preferences can be modeled, as well as the inclusion of government services in the utility function. For the government block, the budget constraint can be expanded to include public employees' wages, as well as transfers for households in the form of financial aid and pensions for retired workers. Another point is to use Bayesian estimation and longer time series in quarterly frequency. It might be challenging because most local government series are in annual frequency and the sample period is short, but it is worth trying.

BIBLIOGRAPHY

ALESINA, A. Austerity: When it works and when it doesn't. Princeton University Press, 2019. Cited on page 97.

ALESINA, A.; FAVERO, C.; GIAVAZZI, F. Effects of austerity: Expenditure-and tax-based approaches. **Journal of Economic Perspectives**, American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203-2418, v. 33, n. 2, p. 141–162, 2019. Cited on page 97.

ALMEIDA, A. N. de et al. **Elasticidades renda e preços: análise do consumo familiar a partir dos dados da POF 2008/2009**. [S.l.], 2011. Cited 2 time(s) on page(s) 61 and 63.

AMATO, J. D.; LAUBACH, T. Rule-of-thumb behaviour and monetary policy. **European Economic Review**, v. 47, n. 5, p. 791–831, 2003. ISSN 0014-2921. Available at: <https://www.sciencedirect.com/science/article/pii/S0014292102002702>. Cited on page 56.

BRAATZ, J. Mega-rs: um modelo de equilíbrio geral dinâmico aplicado ao rio grande do sul. Brasília: Secretaria do Tesouro Nacional, 2018. Cited on page 50.

BRANDÃO, R. V. d. M. O proes e a privatização dos bancos estaduais: o caso do banco do estado do rio de janeiro. **Texto apresentado ao Polis. Niterói**, 2009. Cited on page 87.

BRASIL. National Treasury of Brazil. **Transparent Treasury - Fiscal Statistics and Planning**. Brasília, DF, 2023. Available at: <https://www.tesourotransparente.gov.br/temas/estatisticas-fiscais-e-planejamento>. Accessed on: August 15th, 2023. Cited 2 time(s) on page(s) 61 and 63.

BRASIL. National Treasury of Brazil. **Transparent Treasury - Fiscal Statistics and Planning**. Brasília, DF, 2023. Available at: <https://www.tesourotransparente.gov.br/temas/contabilidade-e-custos/relatorios-contabeis-e-series-historicas-uniao>. Accessed on: August 15th, 2023. Cited 2 time(s) on page(s) 65 and 120.

BRASIL. National Treasury of Brazil. **Accounting and Fiscal Information System of the Brazilian Public Sector**. Brasília, DF, 2024. Available at: <https://siconfi.tesouro.gov.br/siconfi/pages/public/consulta_finbra/finbra_list.jsf>. Accessed on: March 27th, 2024. Cited 7 time(s) on page(s) 61, 62, 63, 64, 65, 120, and 121.

BRASIL. National Treasury of Brazil. **Fiscal Recovery Regime**. Brasília, DF, 2024. Available at: <https://www.tesourotransparente.gov.br/temas/estados-e-municipios/regime-de-recuperacao-fiscal-rrf>. Accessed on: February 19th, 2024. Cited on page 90.

BRASIL. National Treasury of Brazil. **Integrated Vision of Federal, State, Federal District and Municipalities Debt**. Brasília, DF, 2024. Available at: <https://www.tesourotransparente.gov.br/historias/visao-integrada-das-dividas-da-uniao-dos-estados-do-distrito-federal-e-dos-municipios>. Accessed on: January 22th, 2024. Cited on page 86.

BRASIL. Receita Federal do Brasil. **Collected taxes by state**. Brasília, DF, 2024. Available at: <https://dados.gov.br/dados/conjuntos-dados/resultado-da-arrecadacao>. Accessed on: May 14th, 2024. Cited on page 62.

BRAZILIAN INDUSTRIAL CHAMBER OF CONSTRUCTION. **Formal Labor - CAGED**. Rio de Janeiro, 2024. Available at: <http://www.cbicdados.com.br/menu/emprego/>. Accessed on: February 1th, 2024. Cited 2 time(s) on page(s) 73 and 121.

BRINCA, P.; FILHO, J. R. C.; LORIA, F. Business cycle accounting: What have we learned so far? **Journal of Economic Surveys**, n/a, n. n/a, 2023. Available at: <https://onlinelibrary.wiley.com/doi/abs/10.1111/joes.12581>. Cited on page 66.

CARVALHO, C.; NECHIO, F.; TRISTÃO, T. Taylor rule estimation by OLS. **Journal of Monetary Economics**, v. 124, p. 140–154, 2021. ISSN 0304-3932. Available at: <https://www.sciencedirect.com/science/article/pii/S0304393221001203>. Cited 3 time(s) on page(s) 58, 61, and 117.

CARVALHO, J. C. et al. Estimativas do produto potencial, relação capital/produto e depreciação do estoque de capital. Banco Nacional de Desenvolvimento Econômico e Social, 1996. Cited 2 time(s) on page(s) 60 and 63.

CASTRO, M. R. D. et al. Samba: Stochastic analytical model with a bayesian approach. **Brazilian Review of Econometrics**, v. 35, n. 2, p. 103–170, 2015. Cited 4 time(s) on page(s) 20, 43, 62, and 63.

CAVALCANTI, M. A.; VEREDA, L. **Propriedades dinâmicas de um modelo DSGE com parametrizações alternativas para o Brasil**. [S.l.], 2011. Cited 3 time(s) on page(s) 60, 61, and 63.

CAVALCANTI, M. A.; VEREDA, L. Fiscal policy multipliers in a DSGE model for brazil. **Brazilian Review of Econometrics**, v. 35, n. 2, p. 197–232, 2015. Cited on page 25.

CHRISTIANO, L.; EICHENBAUM, M. S.; EVANS, C. **Nominal rigidities and the dynamic effects of a shock to monetary policy**. [S.l.]: National bureau of economic research Cambridge, Mass., USA, 2001. Cited on page 32.

CHRISTIANO, L. J.; EICHENBAUM, M.; EVANS, C. L. Nominal rigidities and the dynamic effects of a shock to monetary policy. **Journal of political Economy**, The University of Chicago Press, v. 113, n. 1, p. 1–45, 2005. Cited on page 28.

CHRISTIANO, L. J.; EICHENBAUM, M. S.; TRABANDT, M. On DSGE models. **Journal of Economic Perspectives**, American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203-2418, v. 32, n. 3, p. 113–140, 2018. Cited 2 time(s) on page(s) 21 and 112.

COSTA-FILHO, J. **Retrieving the state-space representation from Dynare**. [S.l.], 2022. Available at: <https://ideas.repec.org/p/ise/remwps/wp02452022.html>. Cited on page 66.

DAGOSTINI, L. L. M.; DAMMSKI, B. R. Difusão dos modelos DSGE: Uma abordagem bibliométrica. 2022. Cited 2 time(s) on page(s) 23 and 112.

DUBROVSKAYA, J.; SHULTS, D.; KOZONOGOVA, E. Constructing a region DSGE model with institutional features of territorial development. **Computation**, MDPI, v. 10, n. 7, p. 105, 2022. Cited 5 time(s) on page(s) 20, 39, 40, 45, and 48.

ENDERS, W. Applied econometric time series fourth edition. **New York (US): University of Alabama**, 2015. Cited on page 32.

FEDERAÇÃO DAS INDÚSTRIAS DO ESTADO DO RIO GRANDE DO SUL. **Economic Outlook and Perspectives: 2012-2022**. Porto Alegre, 2022. Available at: <https://www.fiergs.org.br/numeros-da-industria/balanco-economico-e-perspectivas>. Accessed on: September 26th, 2023. Cited on page 67.

FEENSTRA, R. C.; INKLAAR, R.; TIMMER, M. P. The next generation of the penn world table. **American economic review**, American Economic Association 2014 Broadway, Suite 305, Nashville, TN 37203, v. 105, n. 10, p. 3150–3182, 2015. Cited on page 60.

FILHO, F. d. H. B.; PESSÔA, S. d. A.; VELOSO, F. A. Evolução da produtividade total dos fatores na economia brasileira com ênfase no capital humano-1992-2007. **Revista Brasileira de Economia**, SciELO Brasil, v. 64, p. 91–113, 2010. Cited 2 time(s) on page(s) 60 and 63.

FLORES, E. M. et al. **International shocks and macroeconomics: a new multi-country DSGE platform for policy analysis in OECD countries.** [S.l.], 2016. Cited 4 time(s) on page(s) 20, 34, 35, and 45.

FOCHEZATTO, A. Construção de um modelo de equilíbrio geral computável regional: aplicação ao rio grande do sul. Instituto de Pesquisa Econômica Aplicada (Ipea), 2003. Cited 2 time(s) on page(s) 50 and 84.

FRISCHTAK, C. R.; MOURÃO, J. Uma estimativa do estoque de capital de infraestrutura no brasil. **Desafios da Nação**, v. 99, 2017. Cited on page 60.

GALÍ, J.; LÓPEZ-SALIDO, J. D.; VALLÉS, J. Understanding the effects of government spending on consumption. **Journal of the european economic association**, Oxford University Press, v. 5, n. 1, p. 227–270, 2007. Cited on page 26.

INSTITUIÇÃO FISCAL INDEPENDENTE. **Analysis of the Fiscal Situation of the Brazilian States - Special Study - Nº 14**. Brasília, DF, 2020. Available at: <https://www2.senado.leg.br/bdsf/bitstream/handle/id/580041/EE14.pdf>. Cited 3 time(s) on page(s) 50, 51, and 87.

INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATÍSTICA. **Broad Consumer Price Index**. Rio de Janeiro, 2023. Available at: <https://www.ibge.gov.br/estatisticas/economicas/precos-e-custos/9256-indice-nacional-de-precos-ao-consumidor-amplo.html?=&t=destaques>. Cited 7 time(s) on page(s) 61, 63, 65, 74, 117, 119, and 120.

INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATÍSTICA. **Quarterly National Accounts System**. Rio de Janeiro, 2023. Available at: <https://www.ibge.gov.br/estatisticas/economicas/contas-nacionais/9300-contas-nacionais-trimestrais.html?=&t=destaques>. Cited 2 time(s) on page(s) 63 and 121.

INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATÍSTICA. **System of Regional Accounts**. Rio de Janeiro, 2023. Available at: <https://www.ibge.gov.br/estatisticas/economicas/contas-nacionais/9054-contas-regionais-do-brasil.html>. Accessed on: October 1th, 2023. Cited 3 time(s) on page(s) 65, 74, and 120.

INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATÍSTICA. **Monthly Commerce Survey**. Rio de Janeiro, 2024. Available at: <https://www.ibge.gov.br/estatisticas/economicas/comercio/9227-pesquisa-mensal-de-comercio.html?=&t=series-historicas>. Accessed on: January 25th, 2024. Cited on page 117.

INSTITUTO BRASILEIRO DE GEOGRAFIA E ESTATÍSTICA. **National Continuous Research by Household Samples**. Rio de Janeiro, 2024. Available at: <https://www.ibge.gov.br/estatisticas/sociais/trabalho/9171-pesquisa-nacional-por-amostra-de-domicilios-continua-mensal.html>. Cited 2 time(s) on page(s) 73 and 121.

INSTITUTO DE PESQUISA ECONÔMICA APLICADA. **Carta de Conjutura - Nº 10**. Rio de Janeiro, 2024. Available at: <https://www.ipea.gov.br/cartadeconjuntura/wp-content/uploads/2024/05/240514_cc_63_nota_10_estoque_de_capital.pdf>. Accessed on: May 14th, 2024. Cited on page 60.

ISSLER, J. V.; PIQUEIRA, N. S. et al. **Estimando a aversao ao risco, a taxa de desconto intertemporal, ea substutibilidade intertemporal do consumo no Brasil usando três tipos de funçao utilidade**. [S.l.], 2001. Cited on page 60.

JUNIOR, C. J. C.; TEIXEIRA, A. M.; SILVA, M. F. da. DSGE para macroeconomia regional: Uma aplicação para o estado de goiás. 2022. Cited 10 time(s) on page(s) 20, 27, 35, 36, 42, 43, 45, 48, 101, and 104.

JUNIOR, L. d. S. M. A política fiscal do estado do rio grande do sul (1970-03): uma abordagem macroeconômica. 2005. Cited on page 87.

JUNIOR, L. d. S. M. A consolidação fiscal do estado do rio grande do sul é necessária? **Indicadores Econômicos FEE**, v. 42, n. 4, p. 85–98, 2015. Cited 2 time(s) on page(s) 87 and 104.

JÚNIOR, L. d. S. M.; JACINTO, P. d. A. Uma retomada da discussão sobre a sustentabilidade da política fiscal do rio grande do sul. **Nova Economia**, SciELO Brasil, v. 21, p. 263–280, 2011. Cited on page 87.

KYDLAND, F. E.; PRESCOTT, E. C. Time to build and aggregate fluctuations. **Econometrica: Journal of the Econometric Society**, JSTOR, p. 1345–1370, 1982. Cited 2 time(s) on page(s) 22 and 65.

LEITE, Á. R. Trajetória da dívida pública no brasil: análise de cenários sob a regra do teto dos gastos públicos (PEC 55/241) dentro da estrutura de um modelo DSGE. Faculdade de Economia, 2019. Cited on page 65.

LIM, D. Small open economy DSGE model with natural disaster and foreign aid. **Policy Brief AKI Res. Grants Econ**, n. 2017, p. 9, 2017. Cited 2 time(s) on page(s) 44 and 45.

LINDÉ, J. DSGE models: still useful in policy analysis? **Oxford Review of Economic Policy**, Oxford University Press UK, v. 34, n. 1-2, p. 269–286, 2018. Cited 2 time(s) on page(s) 31 and 83.

LOPREATO, F. L. C. O endividamento dos governos estaduais nos anos 90. **Economia e Sociedade**, v. 9, n. 2, p. 117–158, 2000. Cited on page 85.

LOPREATO, F. L. C. **O colapso das finanças estaduais e a crise da federação**. [S.l.]: Unesp, 2002. Cited on page 85.

LOPREATO, F. L. C. Um olhar sobre a política fiscal recente. **Economia e Sociedade**, v. 11, n. 2, p. 279–304, 2002. Cited on page 85.

LUZ, I.; VIDEIRA, S. A privatização dos bancos estaduais no brasil. **ENCUENTRO DE**, 2013. Cited on page 87.

MIRKO, A.; SEBASTIAN, W. **Market Regulation, Cycles and Growth in a Monetary Union**. [S.l.], 2019. Cited on page 21.

MONTEIRO, D. **Macrofinancial Dynamics in a Monetary Union**. [S.l.], 2023. Cited on page 21.

NOVA, A. C.; MARQUETTI, A. A. A crise das finanças públicas no rio grande do sul: origem e evolução no período 1999-06. **Ensaios FEE**, v. 30, 2009. Cited on page 87.

OKANO, M. et al. **Development of a Regional DSGE Model in Japan: Empirical Evidence of Economic Stagnation in the Kansai Economy**. [S.l.], 2015. Cited 23 time(s) on page(s) 20, 25, 30, 35, 36, 37, 38, 40, 43, 45, 48, 49, 51, 52, 53, 56, 58, 59, 67, 82, 103, 104, and 118.

PAIVA, W. d. L. Política fiscal e efeitos dinâmicos na economia cearense em uma análise de equilíbrio geral: a importância de investimentos públicos sustentáveis e efetivos. 2019. Cited 13 time(s) on page(s) 20, 27, 35, 36, 40, 41, 42, 43, 45, 48, 96, 100, and 104.

PAPAGEORGIOU, D.; KAZANAS, A. **A Dynamic Stochastic General Equilibrium Model for a Small Open Economy: Greece**. [S.l.], 2013. Cited 9 time(s) on page(s) 19, 24, 31, 34, 45, 49, 83, 100, and 104.

PEIRIS, S.; SAXEGAARD, M. An estimated DSGE model for monetary policy analysis in low-income countries. IMF Working paper, 2007. Cited on page 44.

PENNA, C. et al. Measuring national policy impacts in a region: Samba+reg. **Revistas da USP**, n. v. 53 n. 4, 2023. Cited 4 time(s) on page(s) 20, 43, 45, and 46.

PLOSSER, C. I. Understanding real business cycles. **Journal of Economic Perspectives**, American Economic Association, v. 3, n. 3, p. 51–77, 1989. Cited 3 time(s) on page(s) 23, 84, and 112.

PORSSE, A. A.; PALERMO, P. U.; PORTUGAL, M. S. Exercícios de reestruturação tributária para o rio grande do sul: análise com um modelo de equilíbrio geral computável inter-regional. **Textos para Discussão FEE**, v. 89, 2011. Cited on page 50.

PYTLARCZYK, E. **An estimated DSGE model for the German economy within the euro area**. [S.l.], 2005. Cited 9 time(s) on page(s) 19, 23, 31, 32, 33, 34, 35, 45, and 48.

RIO GRANDE DO SUL (State). Department of Economics and Statistics. **State Annual Accounts**. Porto Alegre, 2022. Available at: <https://dee.rs.gov.br/pib-anual>. Cited 6 time(s) on page(s) 61, 63, 65, 67, 120, and 121.

RIO GRANDE DO SUL (State). Secretary of Finance. **Fiscal Recovery Regime Plan**. Porto Alegre, 2022. Available at: <https://www.rrf.rs.gov.br/upload/arquivos/202205/19140321-prf-rs.pdf>. Cited on page 88.

RIO GRANDE DO SUL (State). Secretary of Finance. **Public Debt Annual Reports: 2009-2022**. Porto Alegre, 2022. Available at: <https://tesouro.fazenda.rs.gov.br/conteudo/15572/relatorio-anual-da-divida-publica-do-rs>. Accessed on: October 7th, 2023. Cited 2 time(s) on page(s) 67 and 89.

RIO GRANDE DO SUL (State). Secretary of Finance. **State Public Debt Annual Report**. Porto Alegre, 2022. Available at: <https://tesouro.fazenda.rs.gov.br/conteudo/18623/relatorio-anual-da-divida-2022>. Accessed on: October 6th, 2023. Cited 2 time(s) on page(s) 49 and 90.

RIO GRANDE DO SUL (State). Secretary of Finance. **Technical Note - 4/2022**. Porto Alegre, 2022. Available at: <https://www.rrf.rs.gov.br/upload/arquivos/202205/19141805-anexo-ii-4-nota-tecnica-004-2022-despesas-pessoal.pdf>. Cited on page 89.

RIO GRANDE DO SUL (State). Secretary of Finance. **Fiscal Recovery Plan of Rio Grande do Sul**. Porto Alegre, 2024. Available at: <https://www.rrf.rs.gov.br/o-plano-de-recuperacao-fiscal>. Accessed on: February, 19h, 2024. Cited 2 time(s) on page(s) 90 and 92.

RIO GRANDE DO SUL (State). Secretary of Finance. **Fiscal Recovery Regime Spreadsheet Forecasts**. Porto Alegre, 2024. Available at: <https://www.rrf.rs.gov.br/upload/arquivos/202205/19141101-anexo-i-planilha-de-projecoes-financeiras.xlsx>. Accessed on: February 7th, 2024. Cited on page 91.

ROMER, D. The new keynesian synthesis. **Journal of economic perspectives**, American Economic Association, v. 7, n. 1, p. 5–22, 1993. Cited on page 23.

SANTOS, D. F. C. dos. **Como a vaca foi pro brejo: 50 anos de finanças estaduais do Rio Grande do Sul (1971-2021): 60 perguntas e respostas**. [S.l.: s.n.], 2022. Cited 2 time(s) on page(s) 87 and 88.

SBORDONE, A. M. et al. Policy analysis using DSGE models: an introduction. **Economic policy review**, v. 16, n. 2, 2010. Cited on page 65.

SMETS, F.; WOUTERS, R. An estimated dynamic stochastic general equilibrium model of the euro area. **Journal of the European economic association**, Oxford University Press, v. 1, n. 5, p. 1123–1175, 2003. Cited on page 32.

TAMEGAWA, K. Two-region DSGE analysis of regionally targeted fiscal policy. **Review of Regional Studies**, v. 42, n. 3, p. 249–263, 2012. Cited 4 time(s) on page(s) 20, 30, 31, and 45.

TAMEGAWA, K. Constructing a small-region DSGE model. **International Scholarly Research Notices**, Hindawi, v. 2013, 2013. Cited 16 time(s) on page(s) 20, 22, 24, 25, 26, 27, 28, 29, 30, 35, 36, 43, 45, 46, 48, and 59.

UHLIG, H. A toolkit for analysing nonlinear dynamic stochastic models easily oxford university press. 1999. Cited on page 39.

VASCONCELOS, J. R. d. Ceará, pernambuco, paraná e rio grande do sul: economia, finanças públicas e investimentos nos anos de 1986-1996. Instituto de Pesquisa Econômica Aplicada (Ipea), 1999. Cited on page 86.

WALUYO, J. **Essays on The Impact of Monetary Policy on Regional and Sectoral Economies in Indonesia**. Phd Thesis (PhD Thesis) — Cornell University, 2017. Cited 6 time(s) on page(s) 20, 23, 38, 39, 45, and 48.

GLOSSARY

Small Region: A region inside a greater region (e.g., a country inside a continent or a state/prefecture (city) inside a country) which is geographically smaller and surely smaller in economic terms as well, i.e. it does not significantly impact the country or continent that it belongs to, but it is affected by its policies. Most smaller regions belong to this definition; however, there are exceptions. For example, California State in the United States, is a smaller region in relation to the country. Still, it might affect the national economy, since it is the highest state GDP in the USA. Another example could be the state of São Paulo, in Brazil. Since it is the wealthiest state of the Brazilian economy (in terms of GDP), it might affect the national economy and be affected by it as well. This concept differs slightly from the one of small open economies in macroeconomics. For that academic subject, the small open economy is "small" in economic terms only, e.g. Brazil, which is huge geographically (of continent proportions), but does not affect the world economy due to its small representation among other countries' wealth (in GDP terms).

DSGE Models: Dynamic macroeconomic models based on general equilibrium identities (microeconomic foundations) which suffer stochastic or deterministic shocks that disturb the equilibrium conditions of the endogenous variables and show the path back to their original point[1].

Regional DSGE Models: A DSGE model applied regionally. This means the model describes a region inside a greater framework, usually a nation or continental economy. Even if it is a closed economy, with no interaction with other regions, it is still influenced by a central authority (divided into central government and a monetary authority), representing the "bigger" region.

[1] Although this explanation is brief, it is necessary to differentiate it from a regional DSGE model. For more information about the history of DSGE modelling, Plosser (1989), Christiano, Eichenbaum and Trabandt (2018) and DAgostini and Dammski (2022) provide significant contextualization of this methodology.

APPENDIX A – LOG-LINEARIZED EQUATIONS OF KANSAI-RS DSGE MODEL

Euler equation for nondurable goods:

$$\hat{C}_t = E_t\hat{C}_{t+1} - \left(\hat{R}_t - E_t\hat{\pi}^C_{t+1}\right) + \frac{\tau^C}{1+\tau^C}\left(E_t\hat{\tau}^C_{t+1} - \hat{\tau}^C_t\right). \tag{59}$$

First-order conditions for durable goods (land and fixed property):

$$\hat{D}_t = \frac{1}{1-\beta\left(1-\delta^D\right)}\left[\left(1-\delta^D\right)\beta E_t\hat{\mu}_{t+1} - \hat{\mu}_t\right]. \tag{60}$$

Private residential investment:

$$\hat{q}_t + \hat{C}_t + \frac{\tau^C}{1+\tau^C}\hat{\tau}^C_t + \psi^D\left(\hat{I}^D_t - \hat{I}^D_{t-1}\right) - \frac{\tau^D}{1+\tau^D}\hat{\tau}^D_t = \hat{\mu}_t + E_t\hat{\mu}_{t+1} + \beta\psi^D\left(E_t\hat{I}^D_{t+1} - \hat{I}^D_t\right). \tag{61}$$

Law of Motion for durable:

$$\hat{D}_t = \left(1-\delta^D\right)\hat{D}_{t-1} + \delta^D\hat{I}^D_t. \tag{62}$$

Labor for non-durable goods:

$$(\eta - l)N_{ss} - l\hat{N}^C = \hat{W}^C_t - \hat{P}^C_t - \frac{\tau^{WC}}{1+\tau^{WC}}\hat{\tau}^{WC}_t - \frac{\tau^C}{1+\tau^C}\hat{\tau}^C_t - \hat{C}_t. \tag{63}$$

Labor for durable goods:

$$(\eta - l)N_{ss} - l\hat{N}^D = \hat{W}^D_t - \hat{P}^C_t - \hat{q}_t - \frac{\tau^{WD}}{1+\tau^{WD}}\hat{\tau}^{WD}_t - \frac{\tau^C}{1+\tau^C}\hat{\tau}^C_t - \hat{C}_t. \tag{64}$$

Relation between non-durable and durable labor supply:

$$\hat{N}^C_t = \hat{N}^D_t + l^{-1}\left(\hat{w}^D_t - \hat{w}^C_t\right) - l^{-1}q_t + l^{-1}\left(\hat{\tau}^{WC}_t - \hat{\tau}^{WD}_t\right), \tag{65}$$

where $\hat{w}^C_t = \hat{W}^C_t - \hat{P}_t$ and $\hat{w}^D_t = \hat{W}^D_t - \hat{P}_t$. Therefore, they are in real terms (constant price level).

Tobin's Q:

$$\hat{Q}_t = -\left(\hat{R}_t - E_t\hat{\pi}_{t+1} + \beta\left[R^K E_t\hat{R}^K_{t+1} + \left(1-\delta^K\right)E_t\hat{Q}_{t+1}\right]\right). \tag{66}$$

Private inventory investment:

$$\hat{I}_t^C = \frac{\beta}{1+\beta} E_t \hat{I}_{t+1}^C + \frac{1}{1+\beta} \hat{I}_{t-1}^C + \frac{\psi^C}{1-\beta} \hat{Q}_t. \tag{67}$$

Law of Motion for Private Stock of Capital:

$$\hat{K}_t = \left(1 - \delta^K\right) \hat{K}_{t-1} + \delta^K \hat{I}_t^C. \tag{68}$$

Law of motion for relative price:

$$\hat{q}_t = \hat{q}_{t-1} + \hat{\pi}_t^D - \hat{\pi}_t^C. \tag{69}$$

New Keynesian Phillips Curve (NKPC):

$$\begin{aligned}
\hat{\pi}_t^C &= \gamma_F^C E_t \hat{\pi}_{t+1}^C + \gamma_b^C \hat{\pi}_{t-1}^C + K_C^1 \hat{\varphi}_t^C - K_C^2 \hat{\tau}_t^F, \\
\hat{\pi}_t^D &= \gamma_F^D E_t \hat{\pi}_{t+1}^D + \gamma_b^D \hat{\pi}_{t-1}^D + K_D^1 \hat{\varphi}_t^D - K_D^2 \hat{\tau}_t^F, \\
\hat{\pi}^C &= \hat{P}_t^C - \hat{P}_{t-1}^C, \\
\hat{\pi}^D &= \hat{P}_t^D - \hat{P}_{t-1}^D.
\end{aligned} \tag{70}$$

Production function of a non-durable goods firm:

$$\hat{Y}_t^C = \hat{A}_t^C + \alpha \hat{K}_{t-1} + \alpha_G \hat{K^G}_{t-1} + (1 - \alpha - \alpha_G) \hat{N}_t^C. \tag{71}$$

Production function of a durable goods firm:

$$\hat{Y}_t^D = \hat{A}_t^D + \hat{N}_t^D. \tag{72}$$

Real marginal cost for non-durable:

$$\hat{\varphi}_t^C = (1 - \alpha - \alpha_G) \hat{w}_t^C + \alpha \hat{r}_t^K - \hat{A}_t^C - \alpha_G \hat{K} g. \tag{73}$$

Labor demand for non-durable:

$$\hat{K}_{t-1} = \hat{N}_t^C + \hat{w}_t^C - \hat{r}_t^K. \tag{74}$$

Real marginal cost for durable:

$$\hat{\varphi}_t^D = -\hat{A}_t^D + \hat{w}_t^D - \hat{q}_t. \tag{75}$$

Local Government budget constraint:

$$\left(\frac{G^L}{G^L+IG^L}\right)\hat{G}_t^L+\left(\frac{IG^L}{G^L+IG^L}\right)\hat{IG}_t^L=(1-\theta_C)\left(\frac{\tau^C C}{G^L+IG^L}\right)(\tau_t^C+P_t^C+C_t-P_t)+ \\ +(1-\theta_C)\left(\frac{\tau^D I^D}{G^L+IG^L}\right)(\tau_t^D+P_t^D+I_t^D-P_t)+(1-\theta_W)\left(\frac{\tau^{WC}w^C N^C}{G^L+IG^L}\right)(\tau_t^{WC}+w_t^C+N_t^C)+ \\ +(1-\theta_W)\left(\frac{\tau^{WD}w^D N^D}{G^L+IG^L}\right)(\tau_t^{WD}+w_t^D+N_t^D)+\left(\frac{TR}{G^L+IG^L}\right)TR_t. \tag{76}$$

Central Government budget constraint:

$$\left(\frac{G^C}{G^C+IG^C}\right)\hat{G}_t^C+\left(\frac{IG^C}{G^C+IG^C}\right)\hat{IG}_t^C=\theta_C\left(\frac{\tau^C C}{G^C+IG^C}\right)(\tau_t^C+P_t^C+C_t-P_t)+ \\ +\theta_C\left(\frac{\tau^D I^D}{G^C+IG^C}\right)(\tau_t^D+P_t^D+I_t^D-P_t)+\theta_W\left(\frac{\tau^{WC}w^C N^C}{G^C+IG^C}\right)(\tau_t^{WC}+w_t^C+N_t^C)+ \\ +\theta_W\left(\frac{\tau^{WD}w^D N^D}{G^L+IG^L}\right)(\tau_t^{WD}+w_t^D+N_t^D)+\theta_F\left(\frac{\tau^F\phi^C}{G^C+IG^C}\right)(\phi_t^C+\tau_t^F)+\theta_F\left(\frac{\tau^F\phi^D}{G^C+IG^C}\right)(\phi_t^D+\tau_t^F)+ \\ +\left(\frac{TR}{G^C+IG^C}\right)TR_t. \tag{77}$$

where $\theta_F=1$, since all firms taxes are for central government, and ϕ^C and ϕ^D are the profits of each sector's firms in real terms, i.e. already divided by prices.

Taylor Rule (Monetary Policy - Central Government):

$$\hat{R}_t=(1-\rho_r-\rho_{r^2})\left(\phi_\pi\hat{\pi}_t^{POA}+\phi_y\hat{Y}_t\right)+\rho_r\hat{R}_{t-1}+\rho_{r^2}\hat{R}_{t-2}. \tag{78}$$

Market Clearing:

$$\hat{Y}_t=\frac{C}{Y}\hat{C}_t+\frac{I^D}{Y}\hat{I}_t^D+\frac{I^C}{Y}\hat{I}_t^C+\frac{G^L}{Y}\hat{G^L}_t+\frac{G^C}{Y}\hat{G^C}_t+\frac{IG^L}{Y}\hat{G^L}_t+\frac{IG^C}{Y}I\hat{G^C}_t. \tag{79}$$

Aggregate prices:

$$\hat{P}_t=(1-\gamma)\hat{P}_t^C+\gamma\hat{P}_t^D. \tag{80}$$

Inflation:

$$\hat{\pi}_t^{POA}=\hat{P}_t-\hat{P}_{t-1}. \tag{81}$$

Aggregate labor:

$$\hat{N}_t=\left(\frac{N^C}{N^C+N^D}\right)\hat{N}_t^D+\left(\frac{N^D}{N^C+N^D}\right)\hat{N}_t^C. \tag{82}$$

Non-Durable and Durable in total output:

$$\hat{Y}_t = (1-\alpha_Y)\hat{Y}_t^C + \alpha_Y \hat{Y}_t^D. \tag{83}$$

Total working earnings of households, representing the sum of labor costs of the firms:

$$\hat{W}_t = (N^C w^C)(\hat{N}_t^C + \hat{w}_t^C) + (N^D w^D)(\hat{N}_t^D + \hat{w}_t^D). \tag{84}$$

Mark II - Inclusion of Public Stock of Capital and Hand-to-Mouth consumers (Non-Ricardians)

Law of Motion of Public Stock of Capital:

$$\hat{K}_t^G = (1-\delta^G \hat{K}_{t-1}^G) + \frac{I^{GL}}{K^G}\hat{I}_t^{GL} + \frac{I^{GC}}{K^G}\hat{I}_t^{GC}. \tag{85}$$

Hand-to-Mouth Consumers (HTM) or "Non-Ricardians":

$$\hat{C}_t^{NR} = NR_1\left(\hat{w}_t^C + \hat{N}_t^C - \frac{\tau^{WC}}{1-\tau^{WC}}\right)\tau_t^{WC} + NR_2\left(\hat{w}_t^D + \hat{N}_t^D - \frac{\tau^{WD}}{1-\tau^{WD}}\right)\tau_t^{WD}, \tag{86}$$

where $NR_1 = \frac{w^C N^C(1-\tau^{WC})}{C^{NR}(1+\tau^C)}$ and $NR_2 = \frac{w^D N^D(1-\tau^{WD})}{C^{NR}(1+\tau^C)}$.

Aggregate Consumption:

$$\hat{C}_t = (1-\zeta)\hat{C}^R + \zeta\hat{C}^{NR}. \tag{87}$$

APPENDIX B – TAYLOR RULE ESTIMATION FOR KANSAI-RS MODEL

Following Carvalho, Nechio and Tristão (2021) in order to estimate the coefficients of Kansai-RS monetary policy rule via OLS Ordinary Least Squares (OLS), we present the results of the autoregressive component (ϕ_r), the elasticity of interest rate to inflation (ϕ_π), and elasticity of interest rate to output gap (ϕ_y) below. Carvalho, Nechio and Tristão (2021) extensively demonstrate that estimating a Taylor Rule using OLS is feasible with little or no loss of efficiency or increase in bias. Following their procedure, which includes the equation and the data used for estimation, we use the basic interest rate of Brazil, known as Selic rate (which corresponds to the FED funds rate in the USA) and provided by the Central Bank of Brazil, the consumer price index of Rio Grande do Sul (RS), represented by the prices of Porto Alegre metropolitan region collected by IBGE (2023a), and a measure of output gap represented by a broad retail sales volume in RS, also collected by IBGE (2024a). The Hodrick-Prescott (HP) filter is applied to the retail sales volume series, resulting in a measure of output gap.

The data for the output gap and inflation are represented by their accumulated moving averages over 12 months. The Selic rate is annualized. The coefficients estimated using monthly or annual data showed virtually the same results for ϕ_π and ϕ_y. ϕ_r monthly estimation is ϕ_r annual estimation raised to the power of twelve. The sample period ranges from 2006 to 2023.

The estimated equation is the following:

$$r_t = \phi_{r_1} r_{t-1} + \phi_{r_2} r_{t-2} + \phi_\pi \pi_t + \phi_y y_t^{gap}. \tag{88}$$

To obtain the real parameter values, we back up the estimation results using the following formulas:

1. $\hat{\phi_r} = \phi_{r_1} + \phi_{r_2}$.
2. $\hat{\phi_\pi} = \frac{\phi_\pi}{1-\hat{\phi_r}}$.
3. $\hat{\phi_y} = \frac{\phi_y}{1-\hat{\phi_r}}$.

Using annual data, $\hat{\phi_r}$ is approximately 0.55 ($\phi_{r_1} = 0.95$ and $\phi_{r_2} = -0.40$), $\hat{\phi_\pi}$ is approximately 1.75 and $\hat{\phi_y}$ is approximately 0.65. $\hat{\phi_\pi}$ is close to national estimated parameters, in line to the fact that RS inflation is about 97% positively correlated to Brazil inflation (Graph 23)[1]. Given that it is reasonable that the Brazilian monetary authority prioritizes control over the output gap, this might explain why RS and Brazilian Taylor Rule coefficients are close to each other. Also, RS's economy is strongly correlated with the national economy in terms of economic activity (not just inflation).

Graph 22 illustrates the regression fit for monthly data. All coefficients (ϕ_{r_1}, ϕ_{r_2}, ϕ_{π}, ϕ_y) are statistically significant at 1% or near it. Only ϕ_y slightly exceeds 0.05 (0.0642) (Table 21). Because Taylor Rule's principle of ϕ_{π} is greater than one is upheld, and estimations using nation-wide data closely align with those using RS data, alongside Okano et al. (2015, page 22), the regional Taylor Rule for RS is justified.

Table 21 – Taylor Rule Estimation for RS - Coefficients Results with monthly data

Variable	Coefficient	Std.Error	t-Statistic	Prob.
Interest (t-1)	1.64983	0.054	30.27	0.0000
Interest (t-2)	-0.665764	0.0537	-12.39	0.0000
Inflation	0.03	0.00786	3.773	0.002
Output Gap	0.01042	0.005598	1.861	0.0642

Source: Author's own research

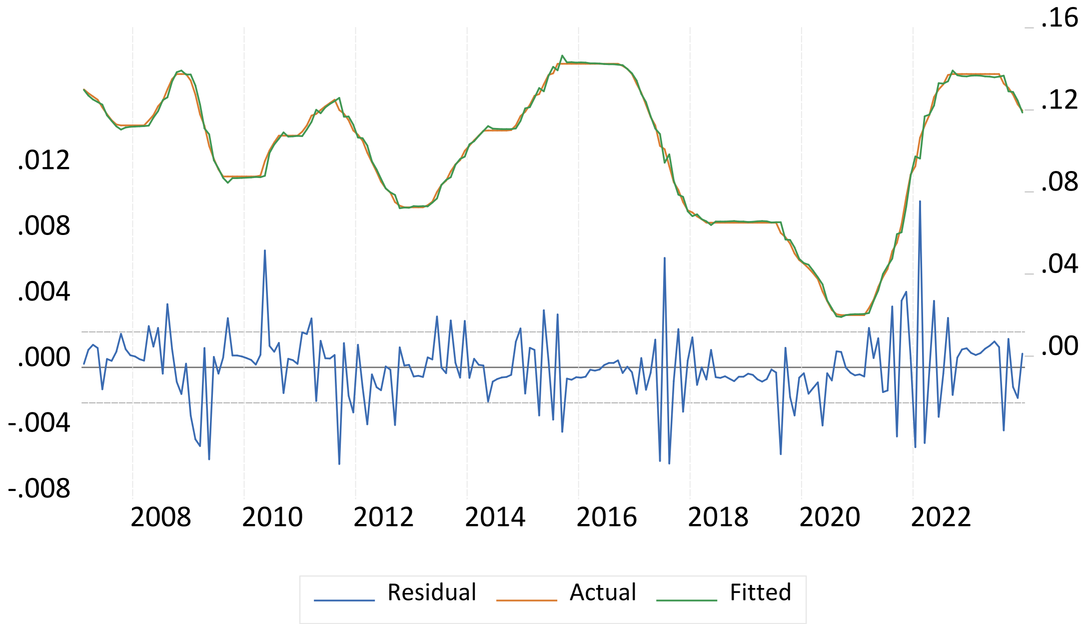
Table 22 – Taylor Rule Estimation for RS - Additional Information for monthly

S.E. of regression	0.002162	Mean dependent var	0.098580
Sum squared resid	0.000926	S.D. dependent var	0.033919
Log likelihood	960.2236	Akaike info criterion	-9.411069
F-statistic	12377.91	Schwarz criterion	-9.329463
Prob(F-statistic)	0.000000	Hannan-Quinn criter.	-9.378055
Durbin-Watson Statistics	2.436657		

Source: Author's own research

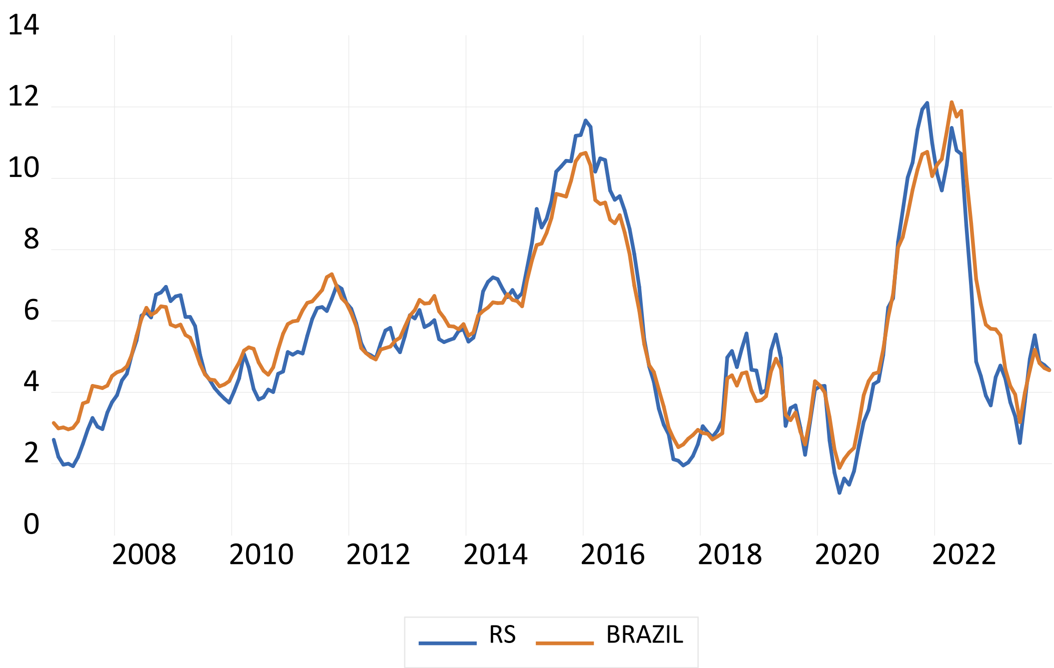
[1] Since Kansai-RS DSGE model is in annual frequency, the monetary parameters were also estimated with annual data. The results 0.55 for $\hat{\phi_r}$, 1.75 for $\hat{\phi_\pi}$ and 0.65 for $\hat{\phi_y}$ are from the annual estimation, which are virtually the same of the quarterly estimation when converted from quarterly to annual frequency.

Figure 22 – Taylor Rule Estimation for RS - Actual, fitted and residual values for monthly data



Source: Author's own research

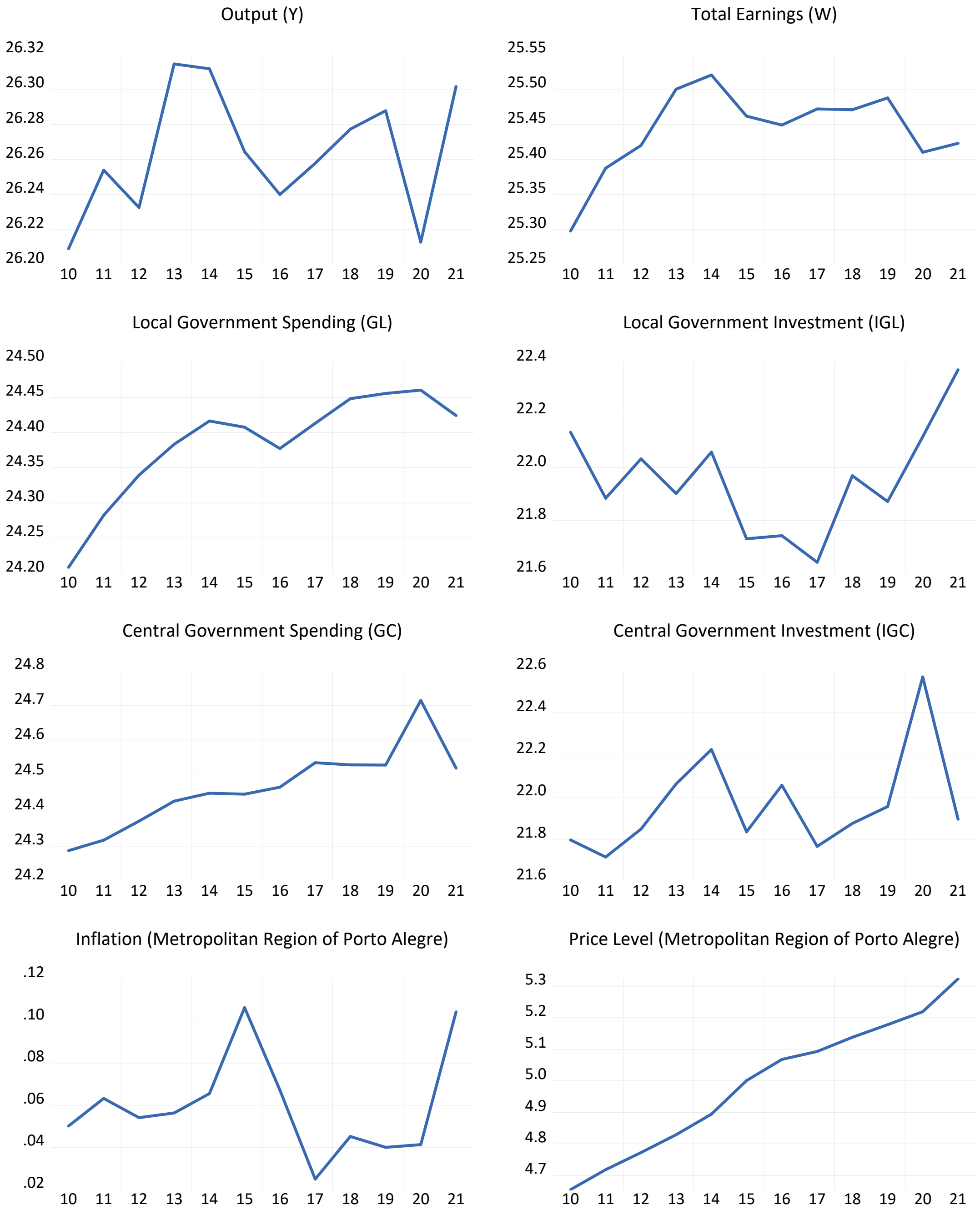
Figure 23 – RS and Brazil's inflation - Compounded 12 month rate of change



Source: IBGE (2023a)

ANNEX A – TIME SERIES OF THE KANSAI-RS MODEL

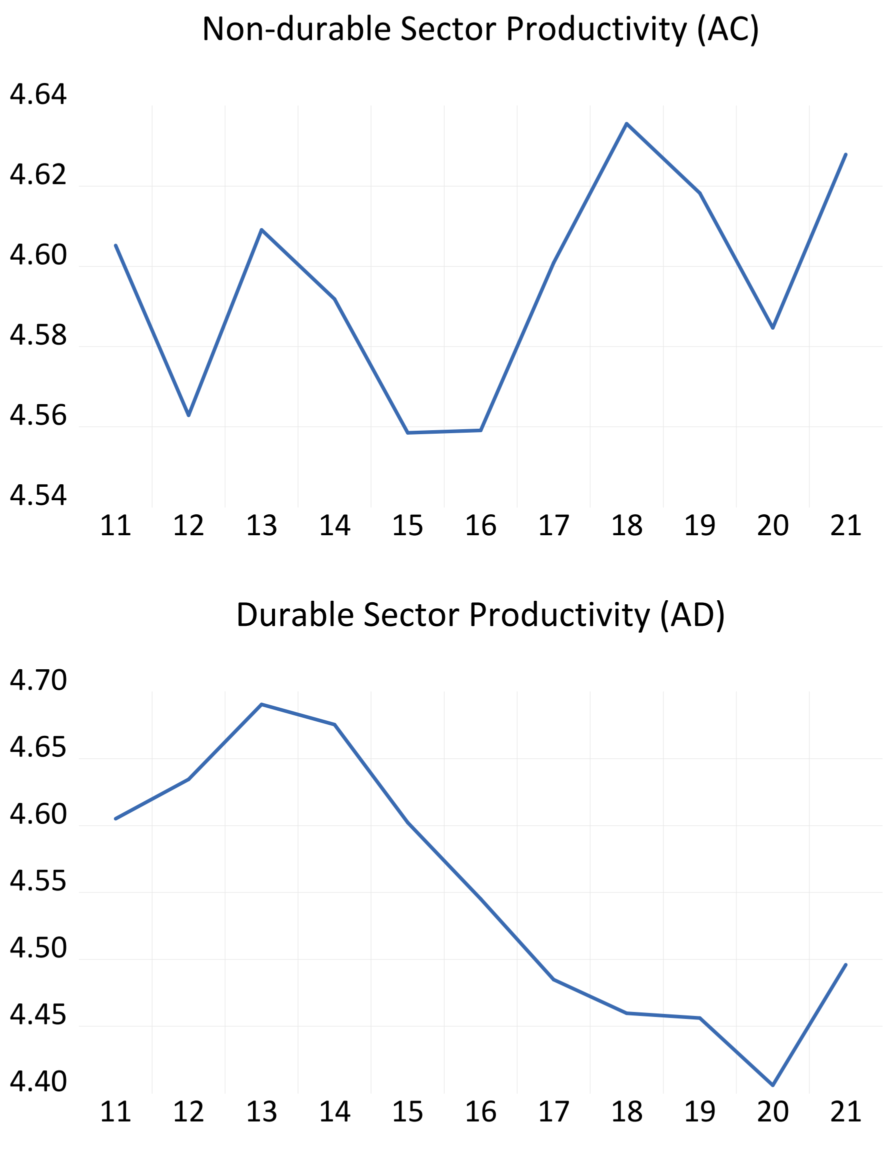
Figure 24 – RS Observed Time Series - 2010 to 2021 (in Natural Log)



Source: DEE/RS (2022), , IBGE (2023c), FINBRA (2024), NTB (2023b) and IBGE (2023a)

ANNEX B – UNOBSERVED TIME SERIES THE OF KANSAI-RS MODEL - PRODUCTIVITY DECOMPOSED USING LABOR, CAPITAL AND OUTPUT GROWTHS

Figure 25 – RS Productivity Series - 2011 to 2021 (in Natural Log)[1]



Source: Author's own research (2024)

[1] Data for RS non-durable and durable employment comes from IBGE (2024b) and CBIC (2024), for investment in stock of capital comes from FINBRA (2024) and IBGE (2023b) (private capital), and for output growth of non-durable and durable sectors comes from DEE/RS (2022).

Pontifícia Universidade Católica do Rio Grande do Sul
Pró-Reitoria de Pesquisa e Pós-Graduação
Av. Ipiranga, 6681 – Prédio 1 – Térreo
Porto Alegre – RS – Brasil
Fone: (51) 3320-3513
E-mail: propesq@pucrs.br
Site: www.pucrs.br